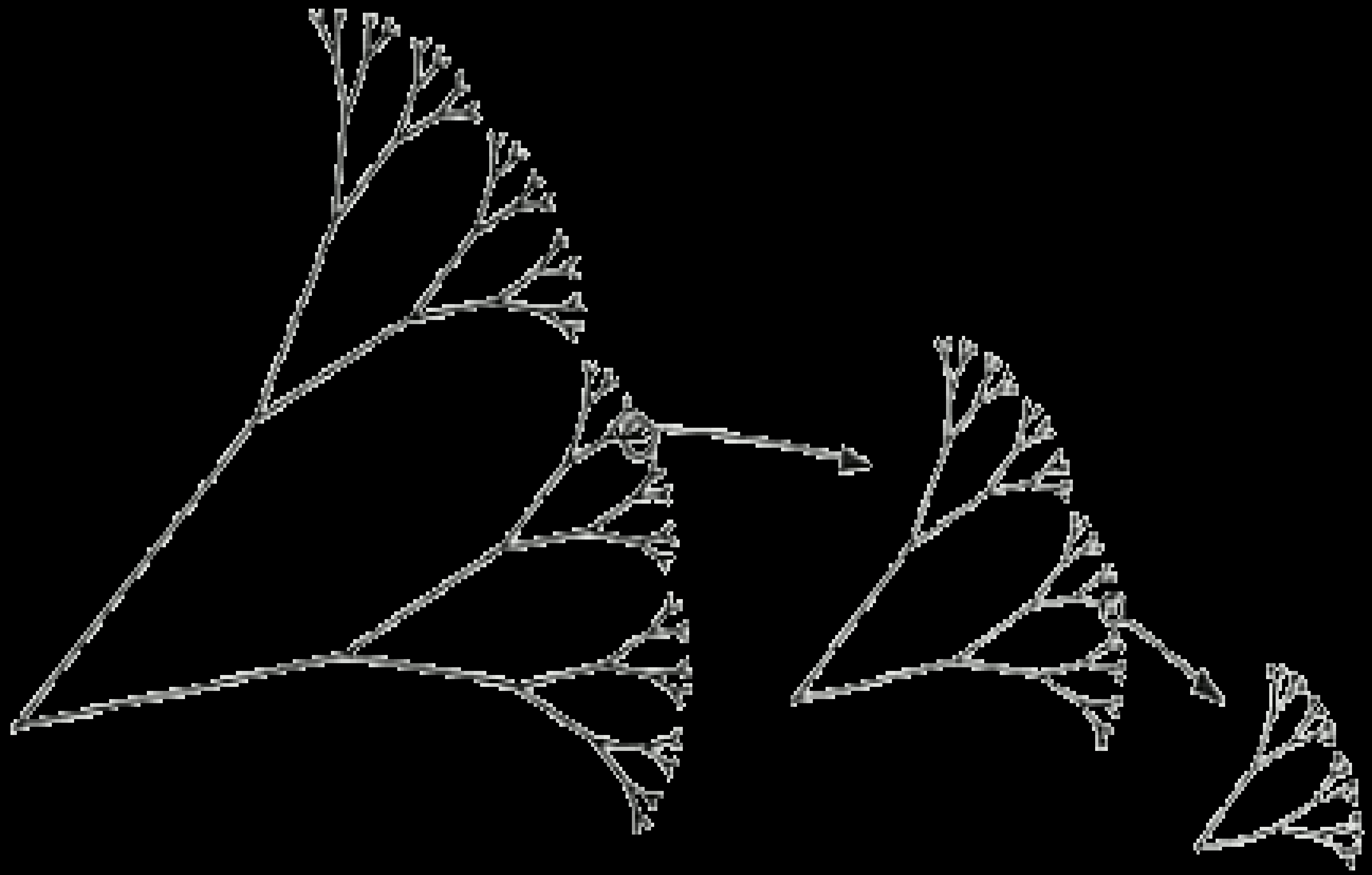
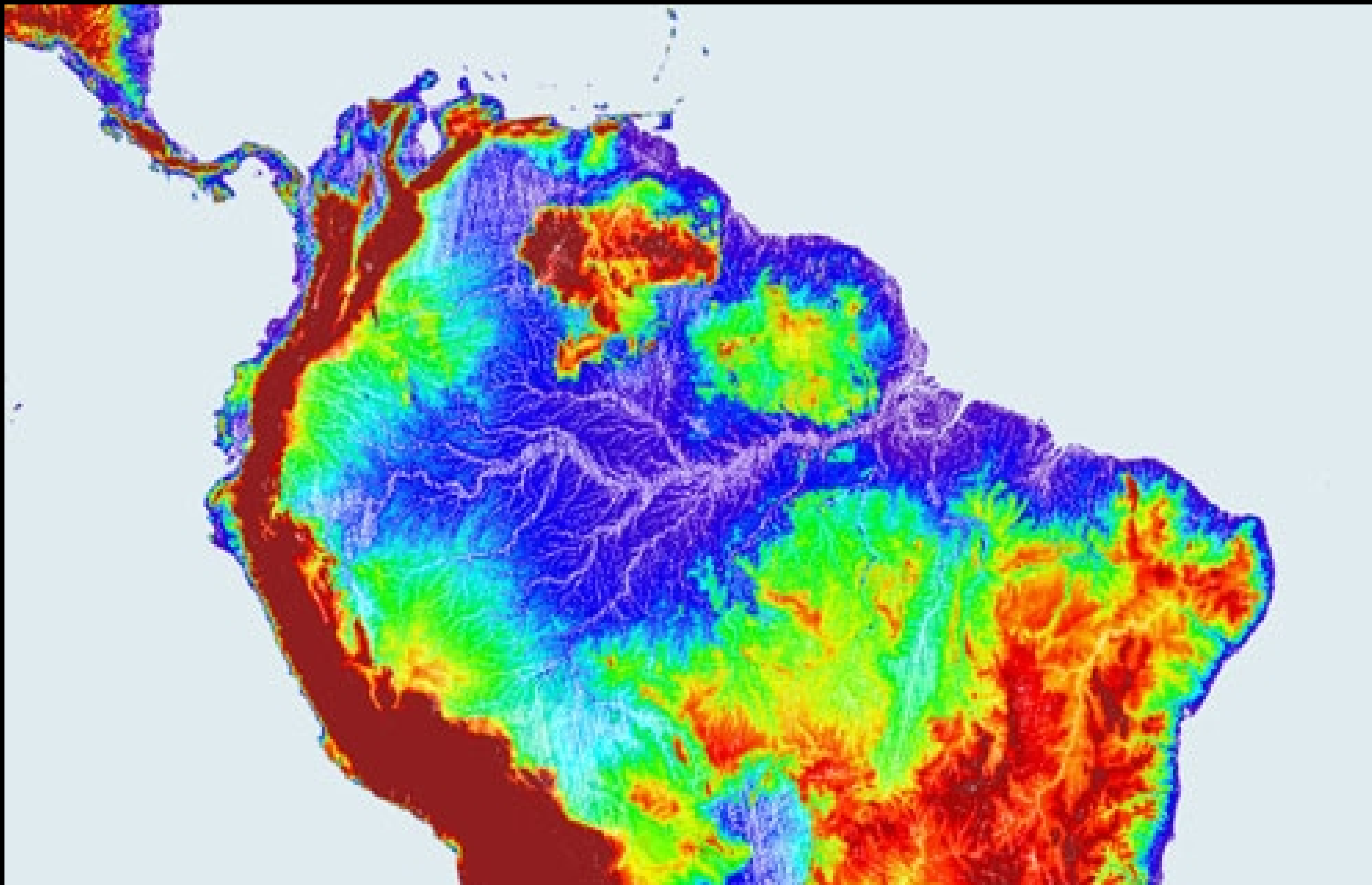
Fractais

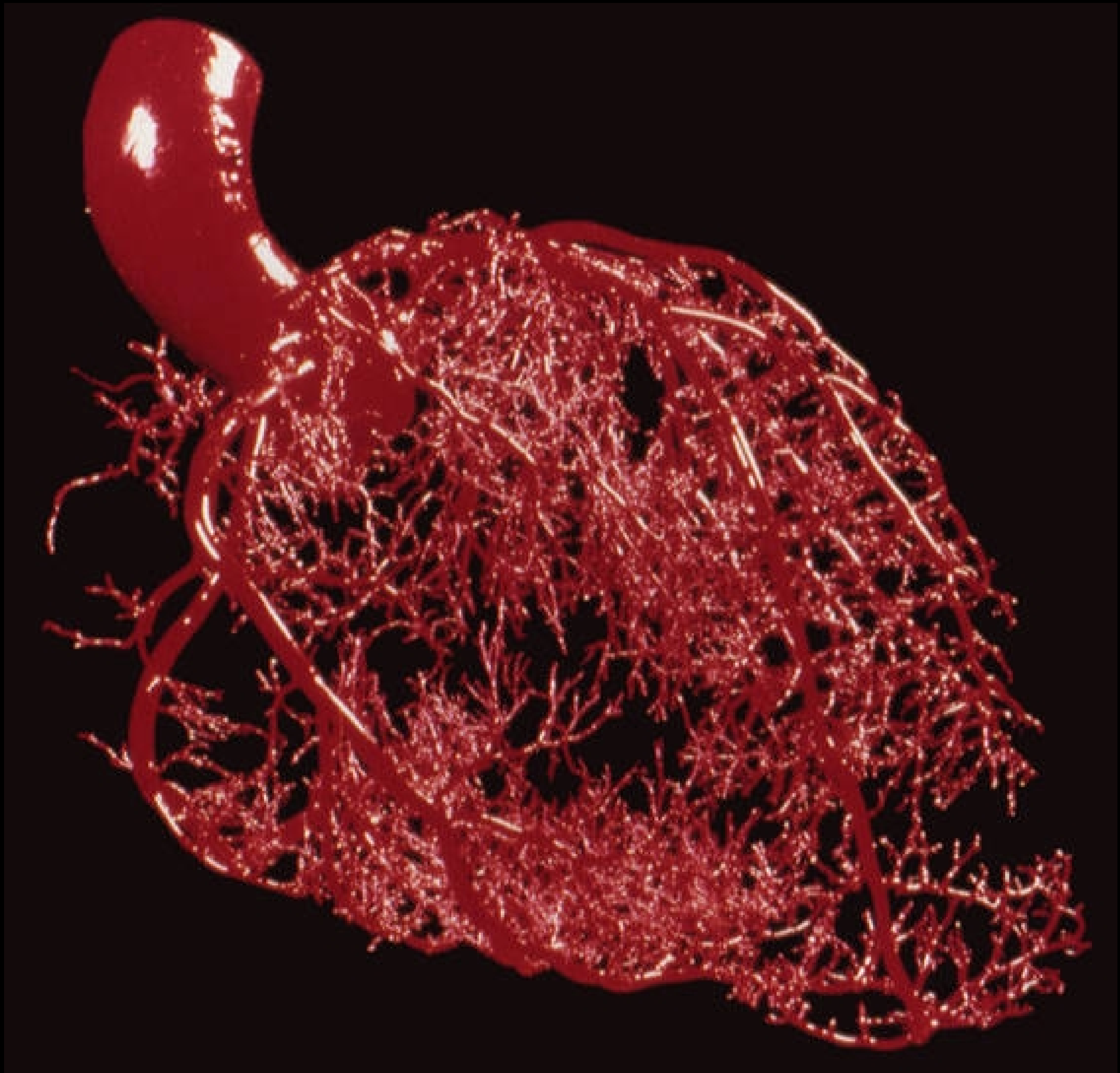
- **Objetos que possuem dimensão fracionária;**

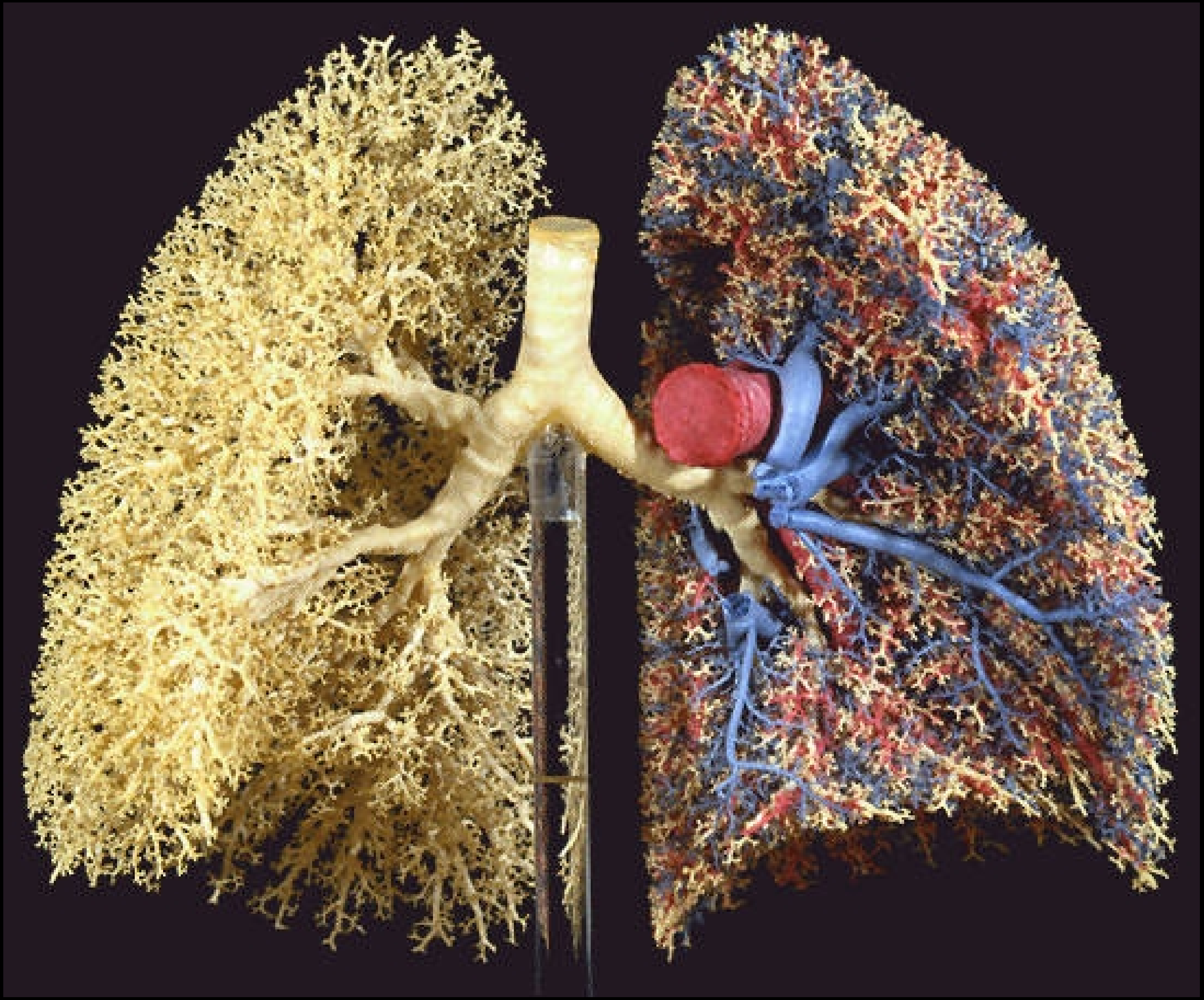
- **Objetos em que as suas partes são semelhantes ao todo;**

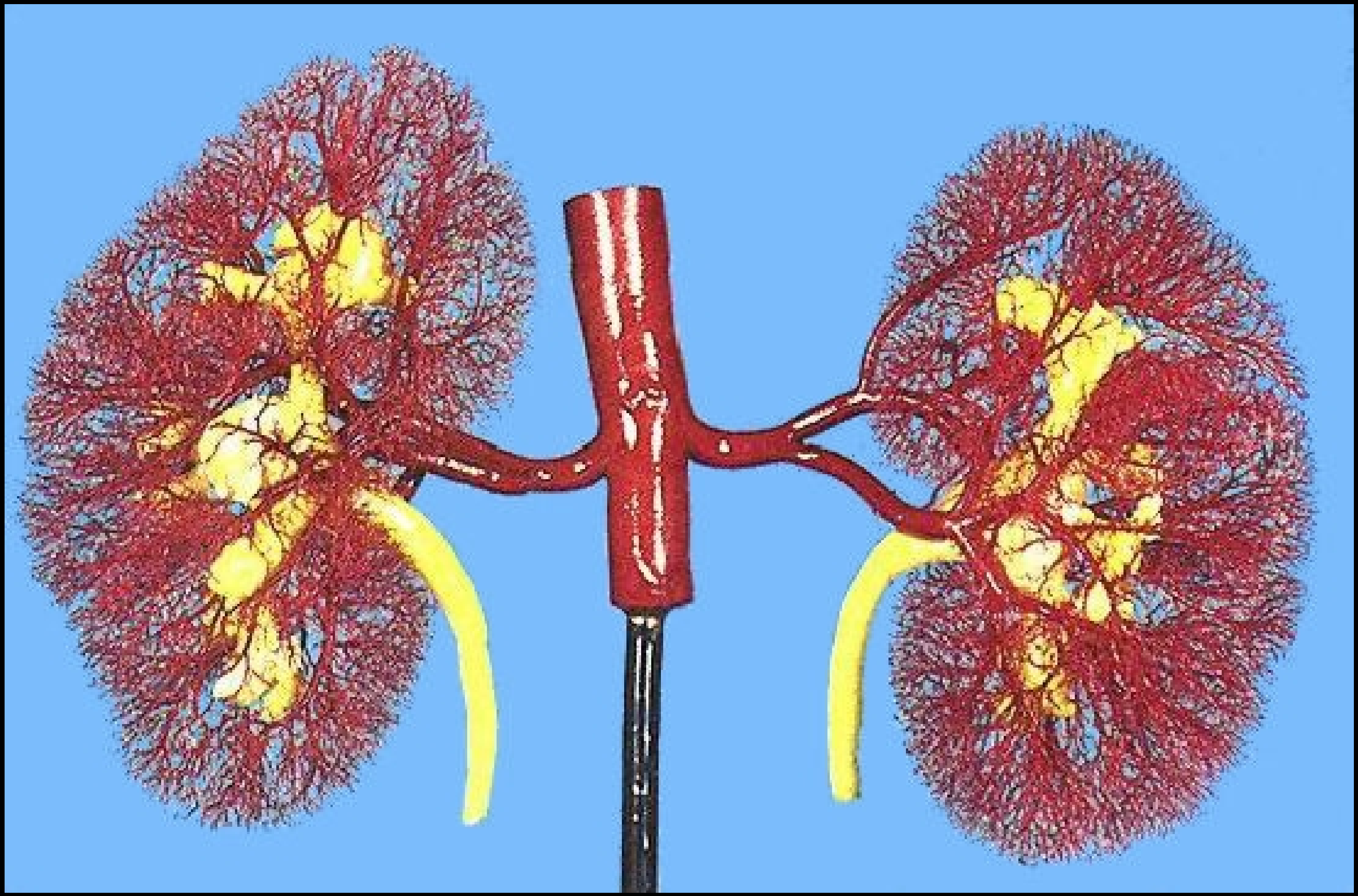


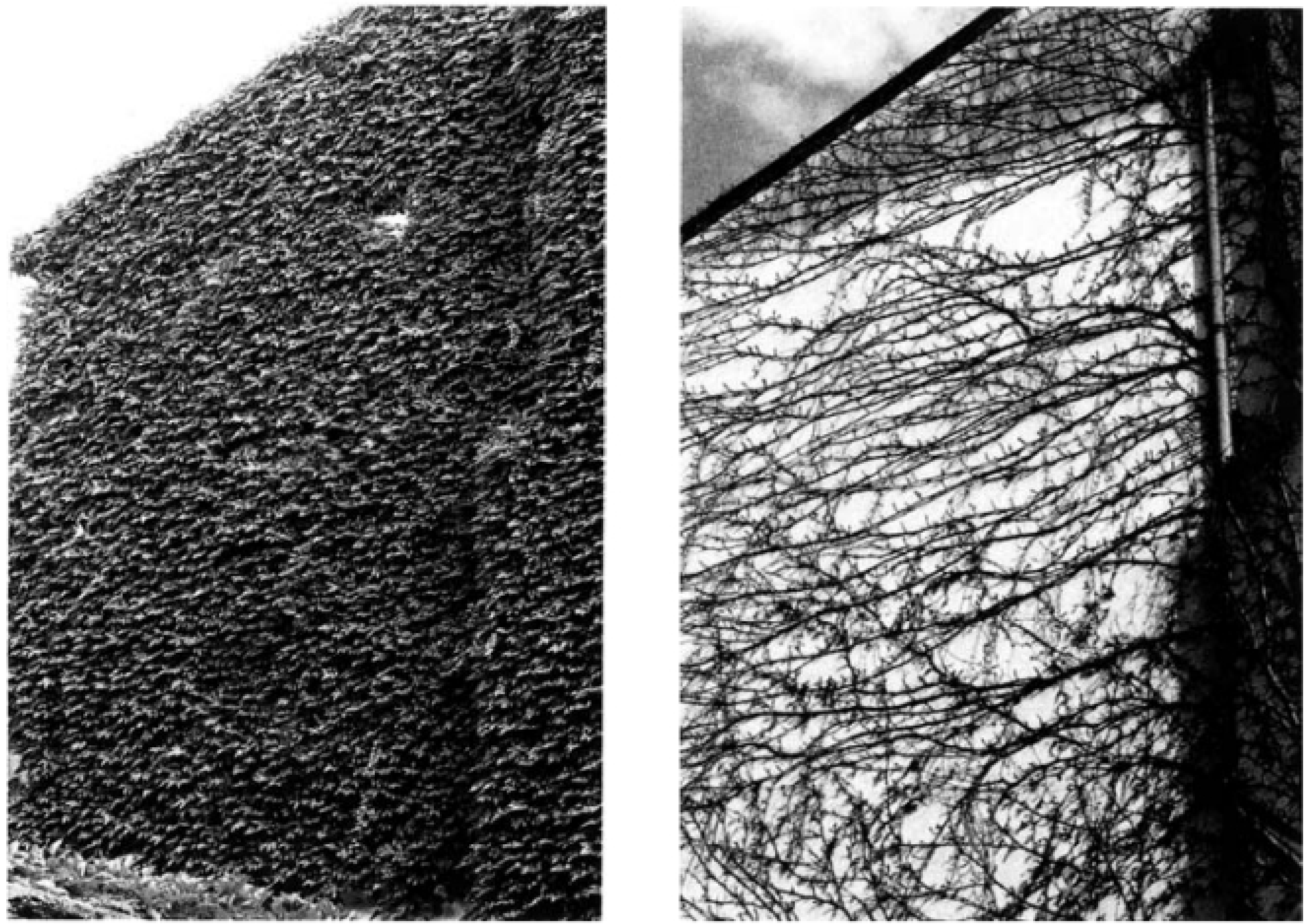
Fractais: formas em que as partes são semelhantes ao todo





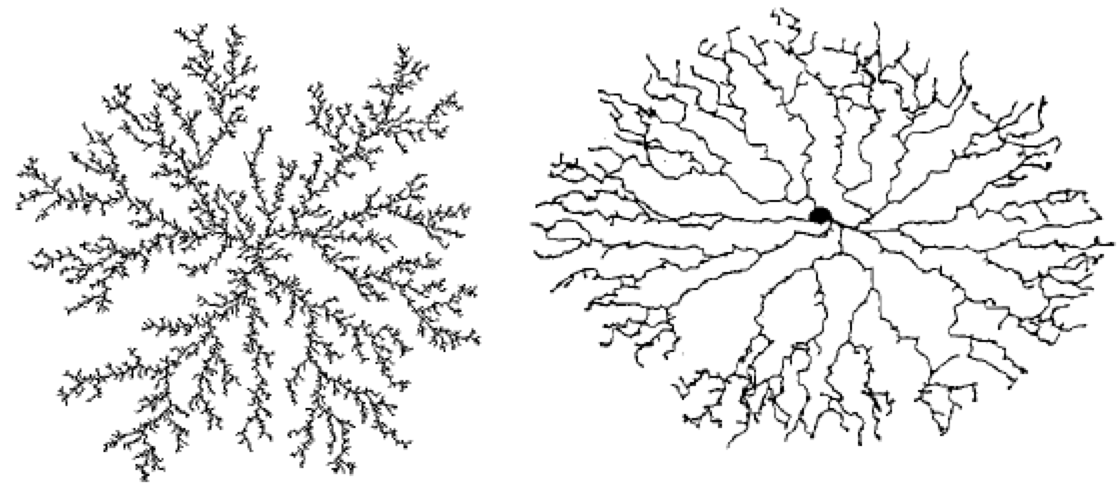






(a) Summer (b) Winter

Figure 14.9. Ivy-covered wall in winter and summer. Is the branch structure in (**a**) fractal or is it space-filling to optimise the leaf coverage (**b**) in the summer?

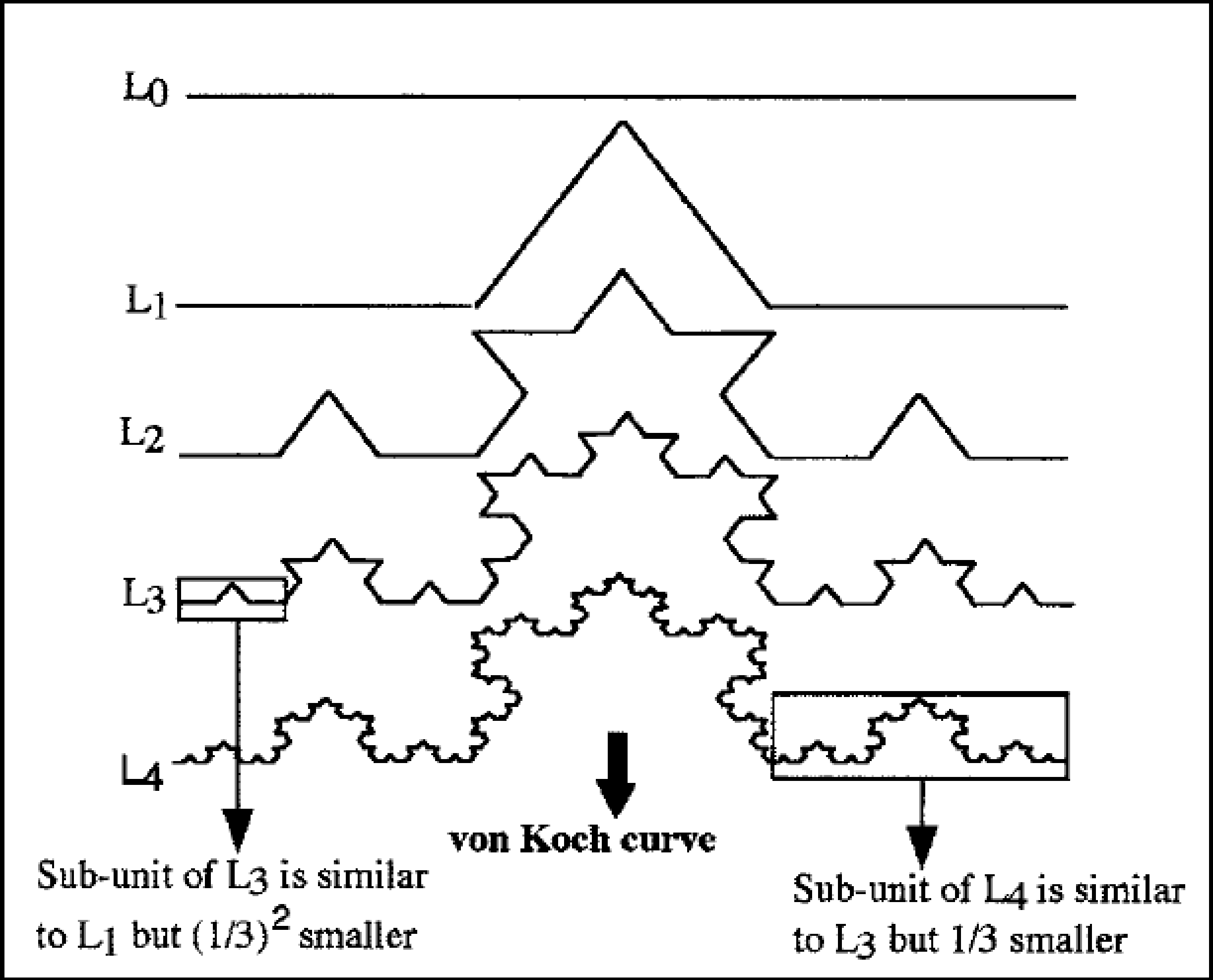


(a) (b)

Figure 14.11. (**a**) Typical pattern generated by diffusion limited aggregation. (From Bassingthwaighte et al. 1994 and reproduced with permission) (**b**) A typical amacrine cell from the central region of the rabbit retina: scale bar is 100 μm (from Figure 14.1). (After Tauchi and Masland 1984) It is premature to draw any conclusions as to the mechanism which forms (**b**) solely by visual comparison between it and the pattern in (**a**).

Objetos Artificiais

Objetos Artificiais



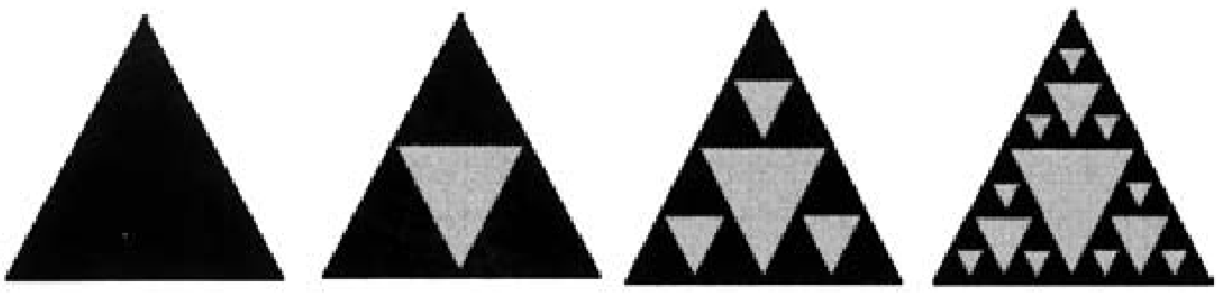
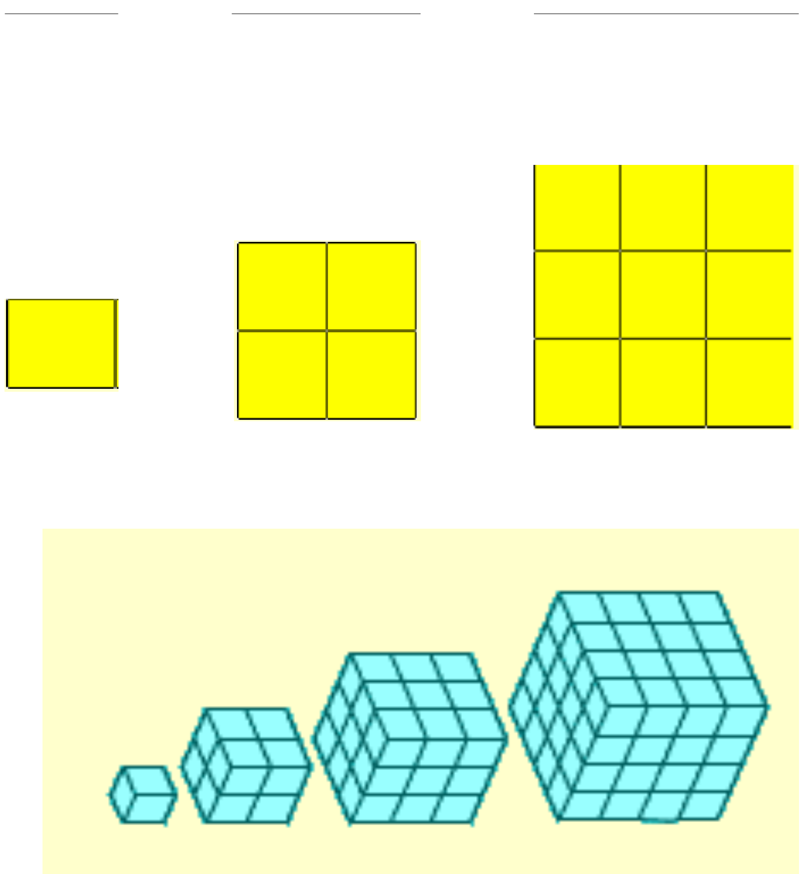


Figure 14.3. Sierpinski triangle fractal. The construction algorithm is to start with a triangle, remove the small inner triangle as in the second figure of four equal triangles and continue in this way with each remaining black triangle but, on a successively reduced scale.

Conceito de Dimensão d



$N = L^1$

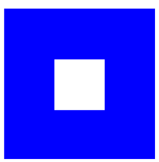
$N = L^2$

$N = L^3$

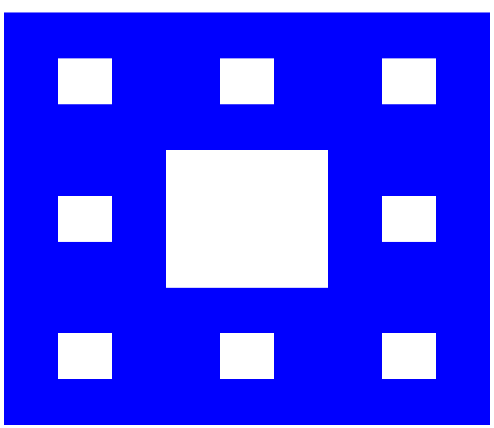
$$N = L^d \quad \text{ou} \quad d = \log(N)/\log(L)$$

Tapete de Sierpinski

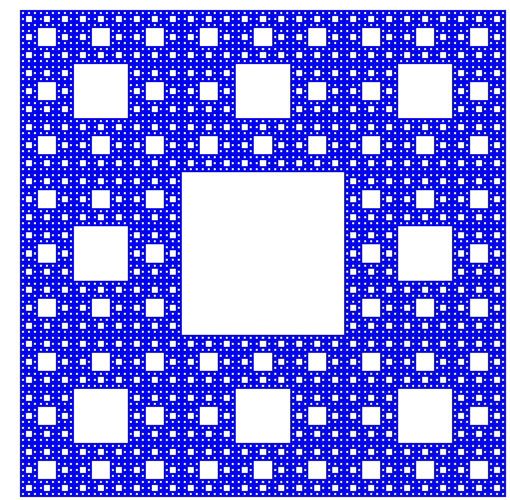
L=1 , N = 1



L = 3 , N =8



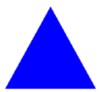
L= 9 , N= 64



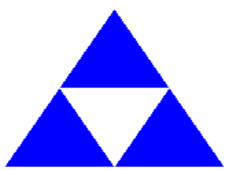
Df =log(N)/log(L)

Df = 1,893...

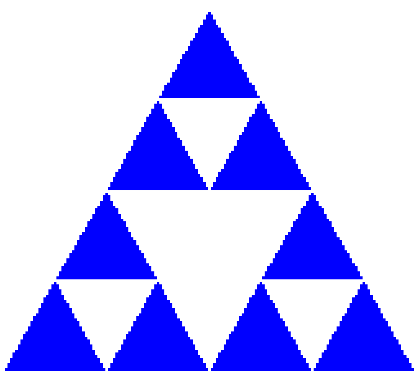
Triangulo de Sierpinski

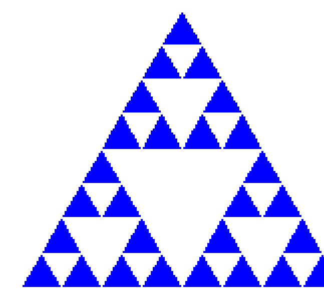


L=1 , N = 1

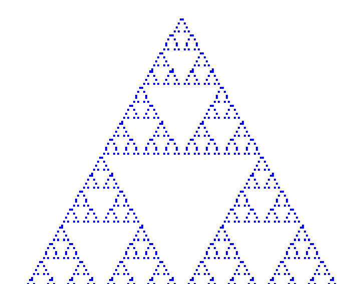


L = 2 , N =3





L= 4 , N= 9



Df = log(N)/log(L)

Df = 1,585...

Dimensões fractais de alguns objectos naturais

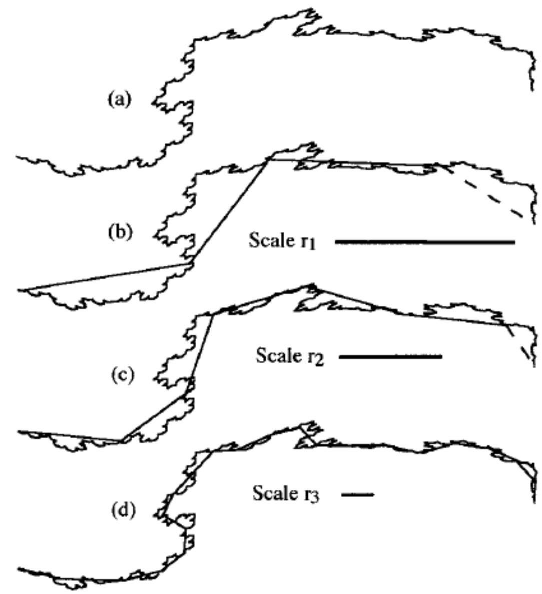
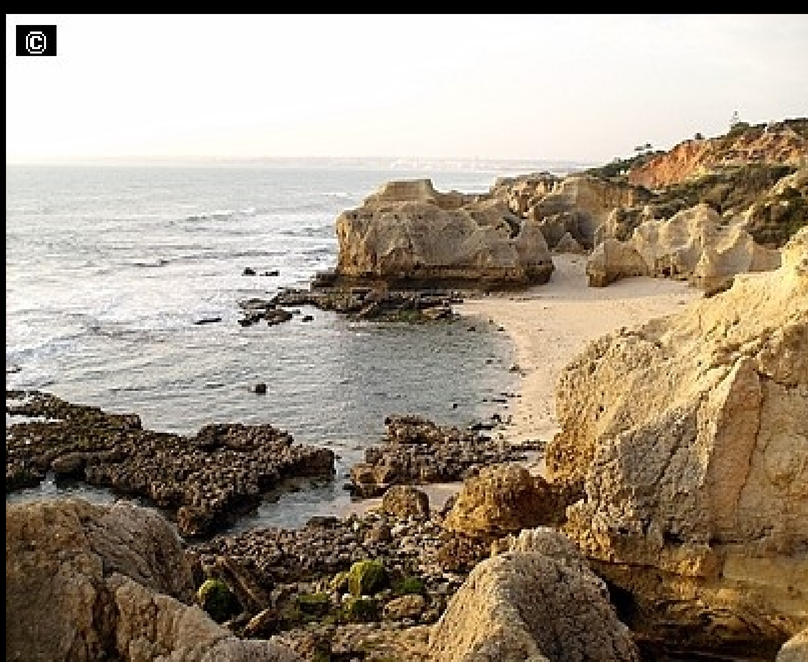
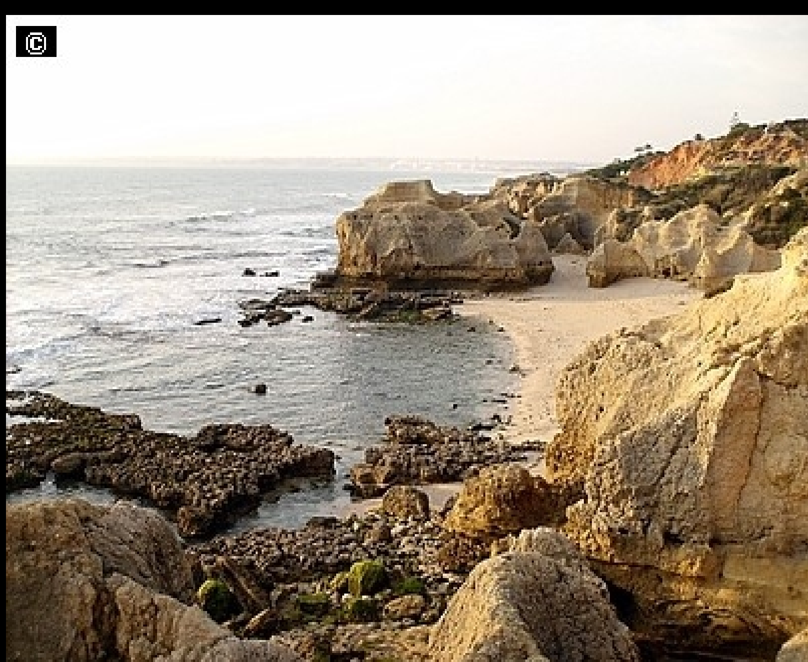


Figure 14.5. The effect on the measured 'length' of the curve in (**a**) as we vary the scale on which we measure it. The 'length' of the curve measured with a large unit of scale, *r* (that is, small magnification) as in (**b**), is smaller than the 'length' measured with the scales in (**c**), which in turn is smaller than that with the scale in (**d**).





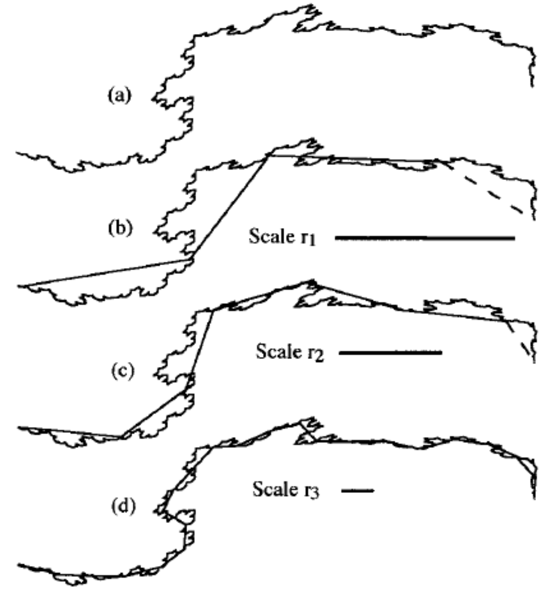
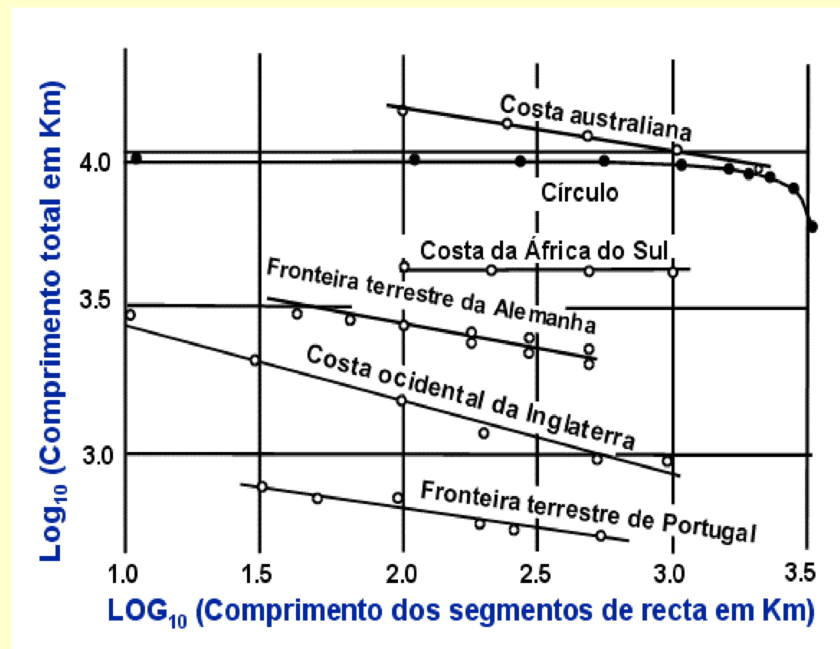


Figure 14.5. The effect on the measured 'length' of the curve in (**a**) as we vary the scale on which we measure it. The 'length' of the curve measured with a large unit of scale, *r* (that is, small magnification) as in (**b**), is smaller than the 'length' measured with the scales in (**c**), which in turn is smaller than that with the scale in (**d**).

- A dimensão fractal da fronteira terrestre de Portugal é 1.14. [Mandelbrot, *Science*, 636–638 (1967)].



Richardson 1961 The problem of contiguity: An Appendix to Statistics of Deadly Quarrels General Systems Yearbook 6:139-187.

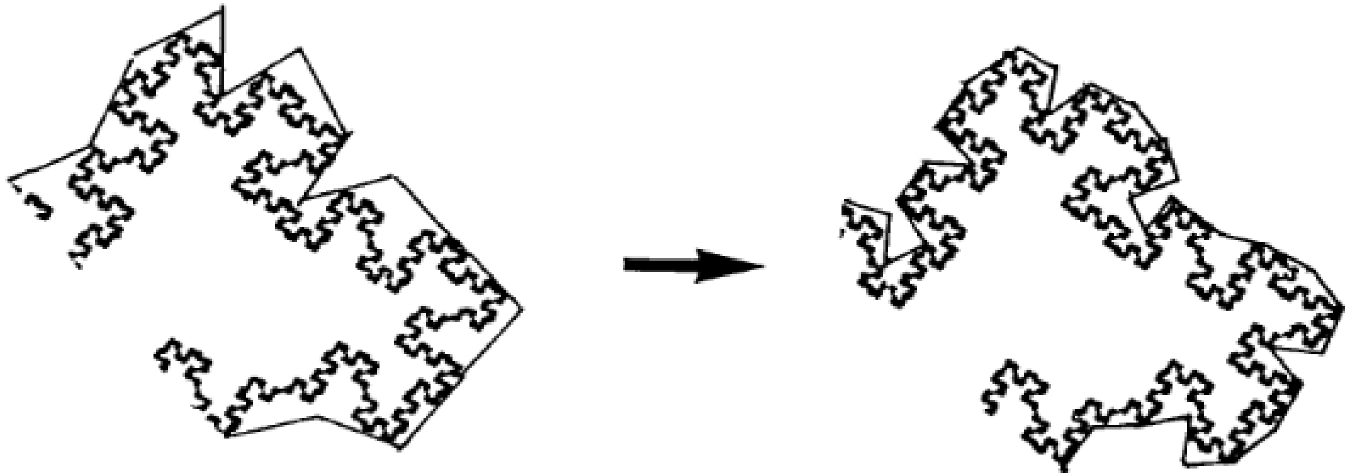


Fig. 2.4. Estimation of the total length of the curve shown in Fig. 2.3C. The total length $L(\epsilon)$ is estimated by covering the curve with an equal-sided polygon with side length ϵ; in successive approximations the length of ϵ decreases.

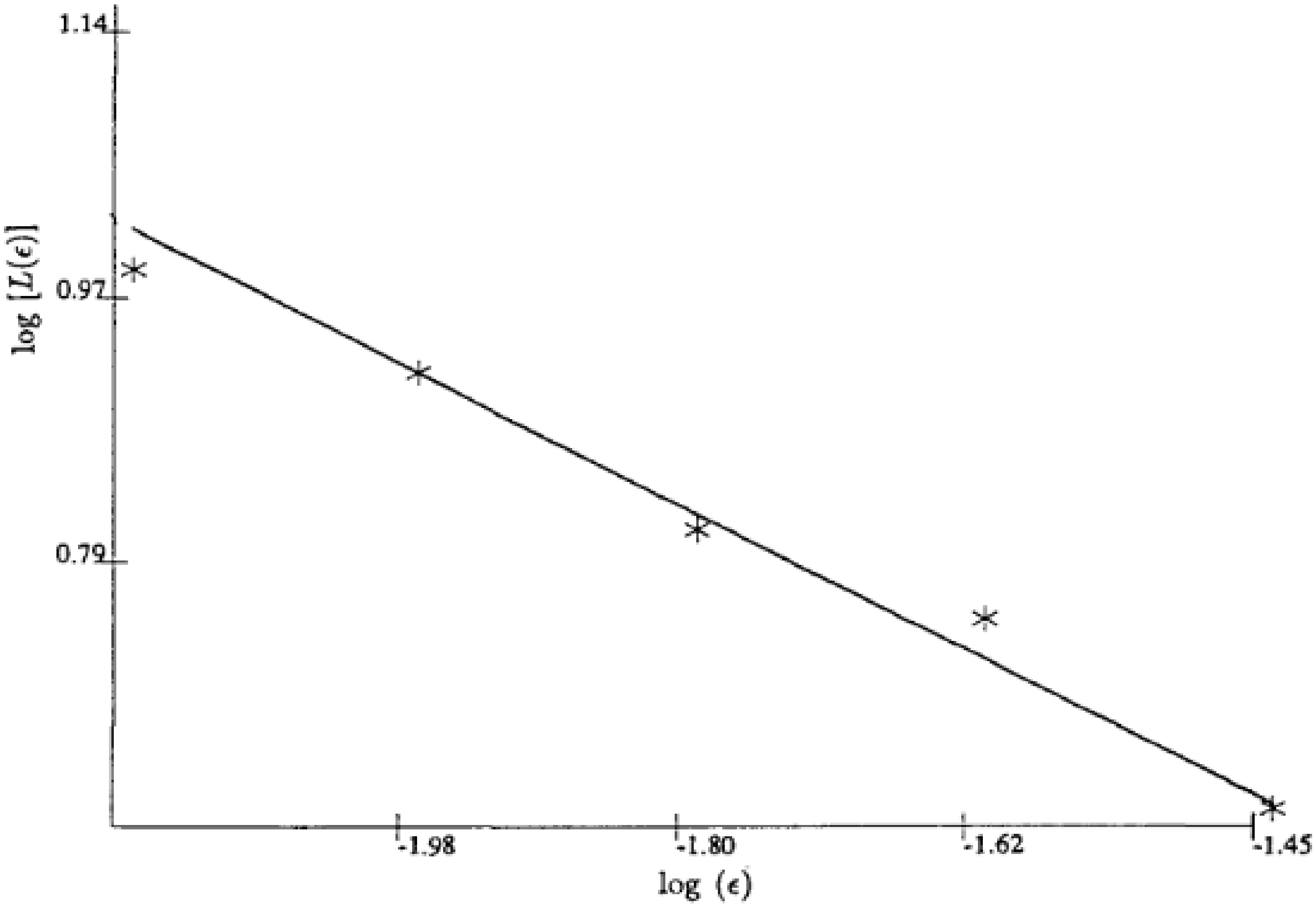
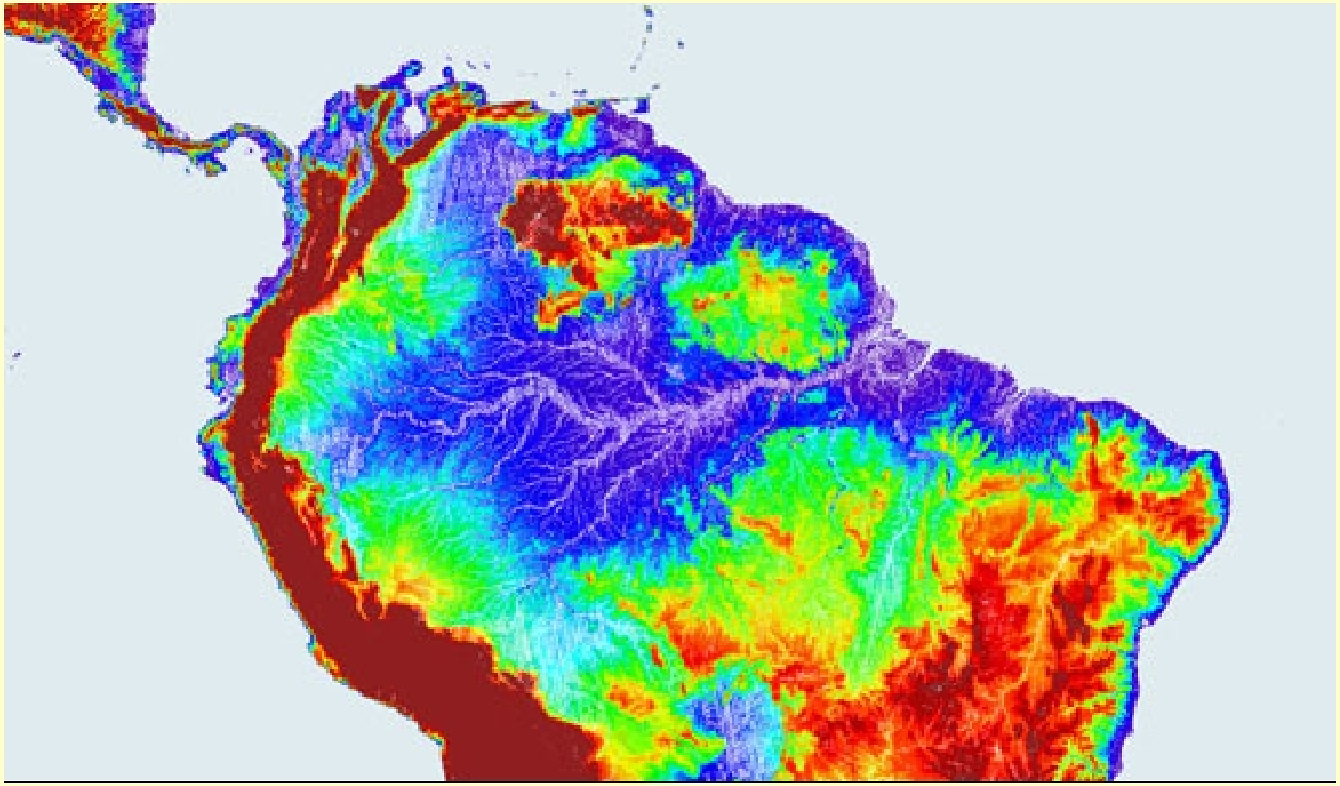


Fig. 2.5. Relation between the total length $L(\epsilon)$ and ϵ, where the value of $L(\epsilon)$ was estimated with the method displayed in Fig. 2.4

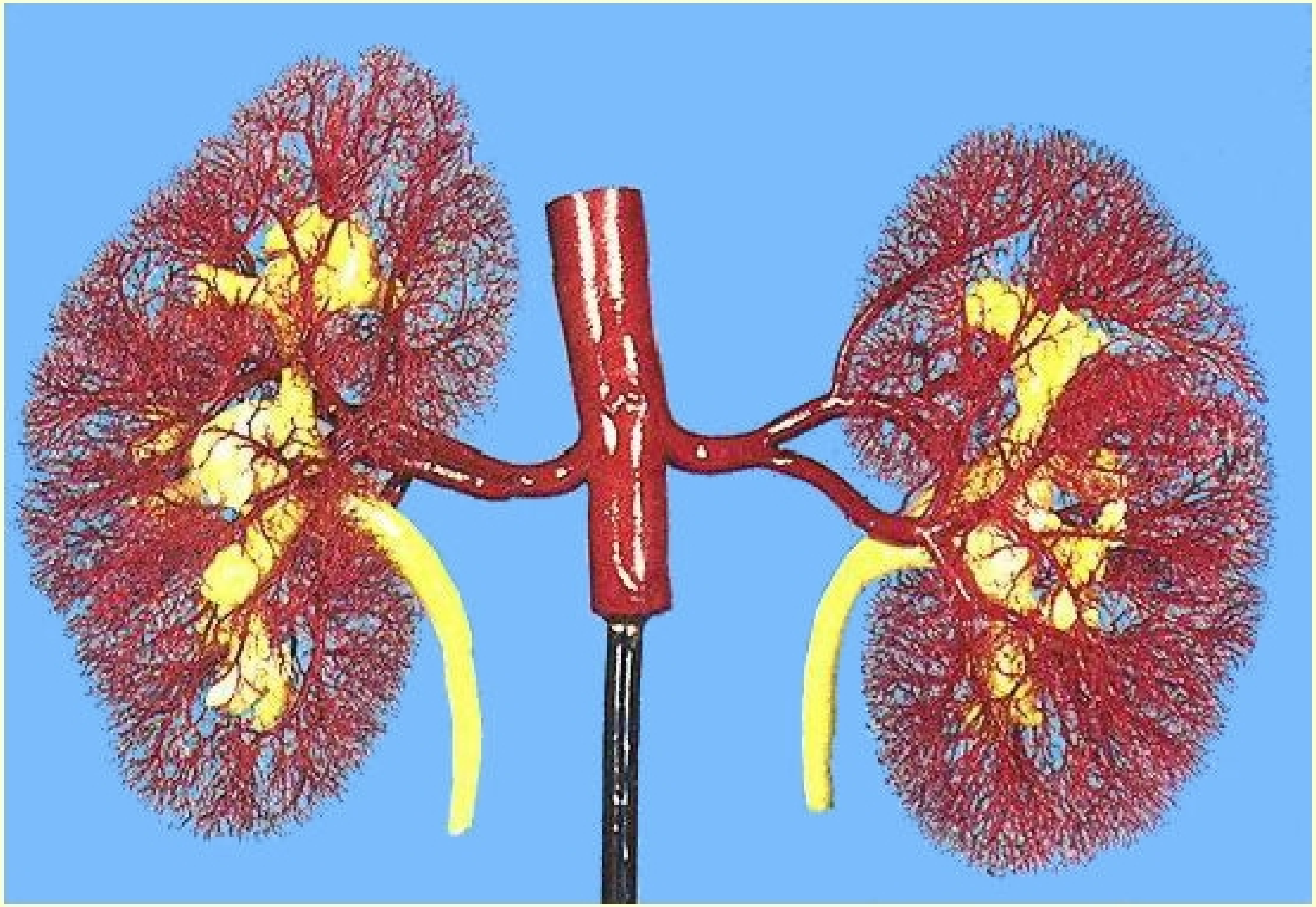
- A dimensão fractal do sistema fluvial do rio Amazonas é 1.85. [H. Takayasu, *Fractals in the Physical Sciences*, Manchester Univ. Press (1990)].



Bacia do rio Amazonas obtida pelo radar de altimetria ERS-1. Cortesia da ESA

- A dimensão fractal dos relâmpagos (no espaço tridimensional) é 1.51. [J. Sañudo et al, *Fractal Dimension of Lightning Discharge*, Nonlinear Processes in Geophysics (1995)].

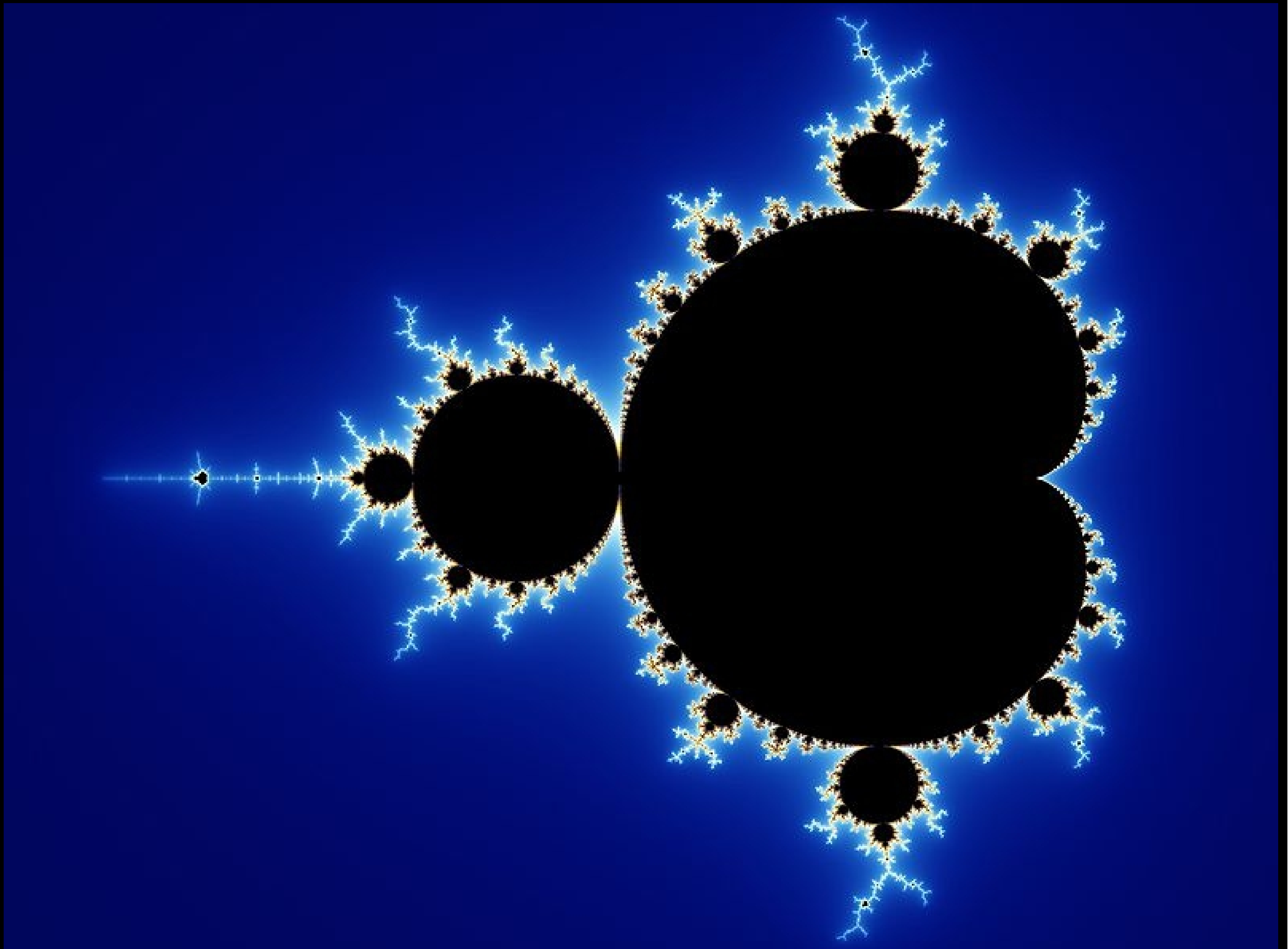
- A dimensão fractal dos angiogramas dos rins (radiografia dos vasos sanguíneos dos rins) é 1.61. [Cross et al, *Quantitation of the renal arterial tree by fractal analysis*, J. Pathol 170:. 479–484 (1993)].

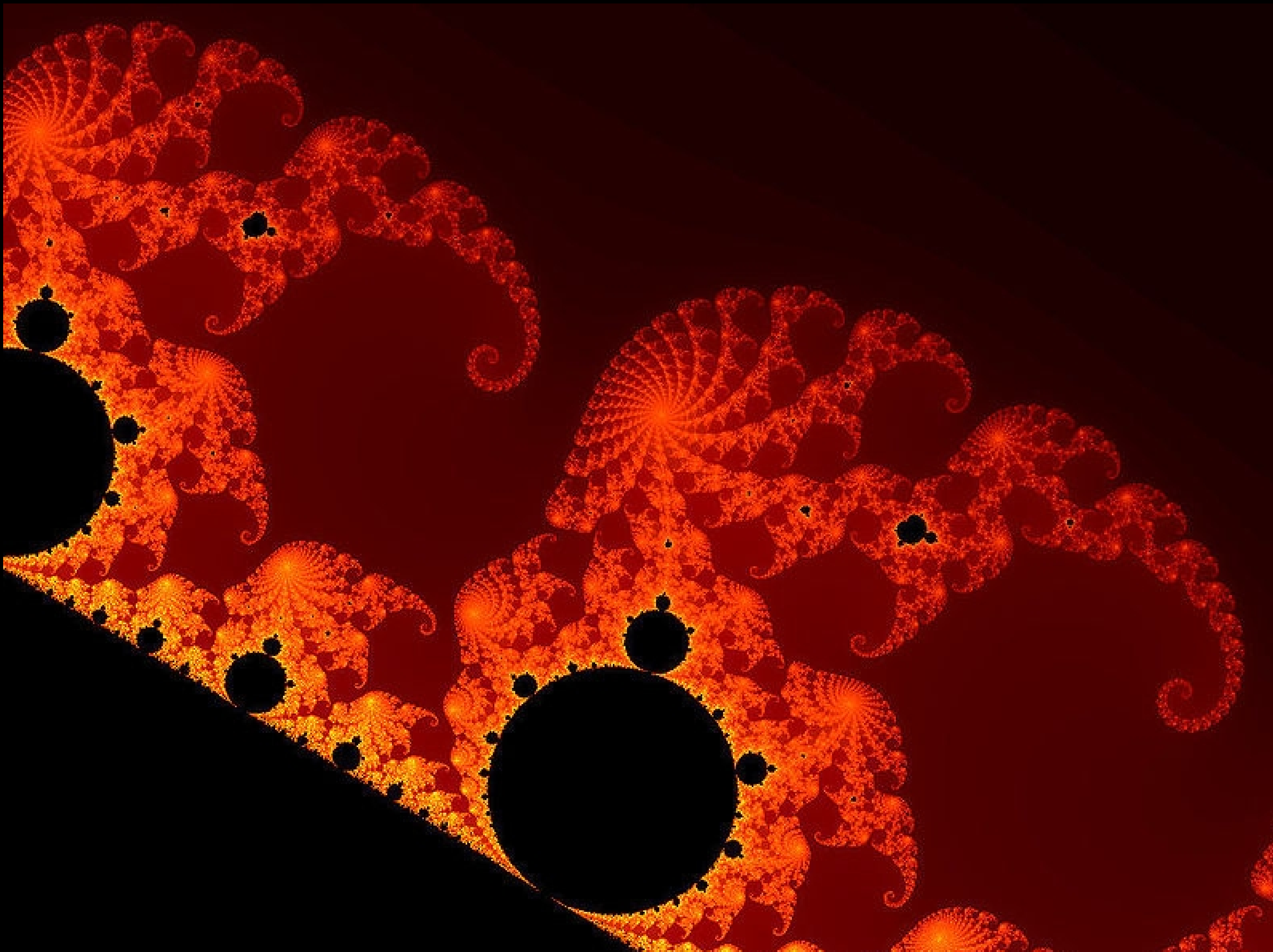


Sistema arterial dos rins. Origem: Gray's Anatomy, 35ª Ed. pag. 1327.

Conjunto de Mandelbrot

$$z = z^2 + c$$





(M) for such crumpled paper balls are quite well described by $L = kM^{1/D}$. D is interpreted as the fractal dimension of the balls and $k \sim (1/\rho)^{1/D}$ is a measure of the average mass-density ρ on these fractal structures. The values obtained for D and k were $D = 2.51 \pm 0.19$, $k = 5.75 \pm 0.71$ for writing paper of surface density $\sigma \sim 80$ g/m^2. The fractal dimension D in this case tells about the complexity or degree of contortion of the area, since a fixed measure of rounded smooth area can enclose a larger volume than a complicated one can. The values of D and k are statistically independent of the students' weight and height. It is experimentally evident that k (lacunarity[1]) has a percent mean square deviation approximately two times larger than that of D, and $(\Delta\rho/\rho) = D(\Delta k/k) \simeq 0.31$. These values show that D is much less affected by the way of crumpling (pressure applied, haste or not, etc.) than the density is. The topological dimension of these balls is $D_T = 2$, since they are made of sheets of paper, which conform with $D_T = 2$. On the other hand, they are embedded in the Euclidean three-dimensional space ($E = 3$), so their fractal D satisfies $D_T = 2 \leqslant D \leqslant E = 3$. In Fig. 3 we give a diagram indicating the frequency distribution of the Ds obtained in the last semester with 89 students. From the geometrical point of

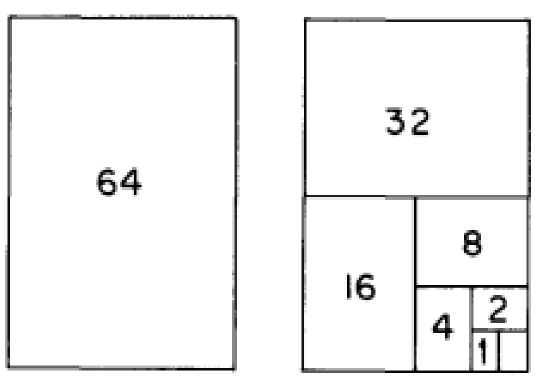


Fig. 1. Dividing two sheets of paper to generate the crumpled paper balls shown in Fig. 2. In this case $n = 6$ (see text).

Fig. 2. A typical set of crumpled paper balls with masses 1,2,4,...,64.

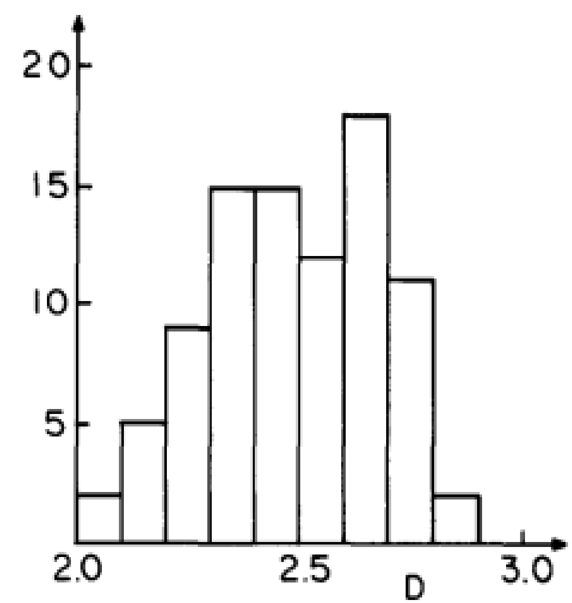
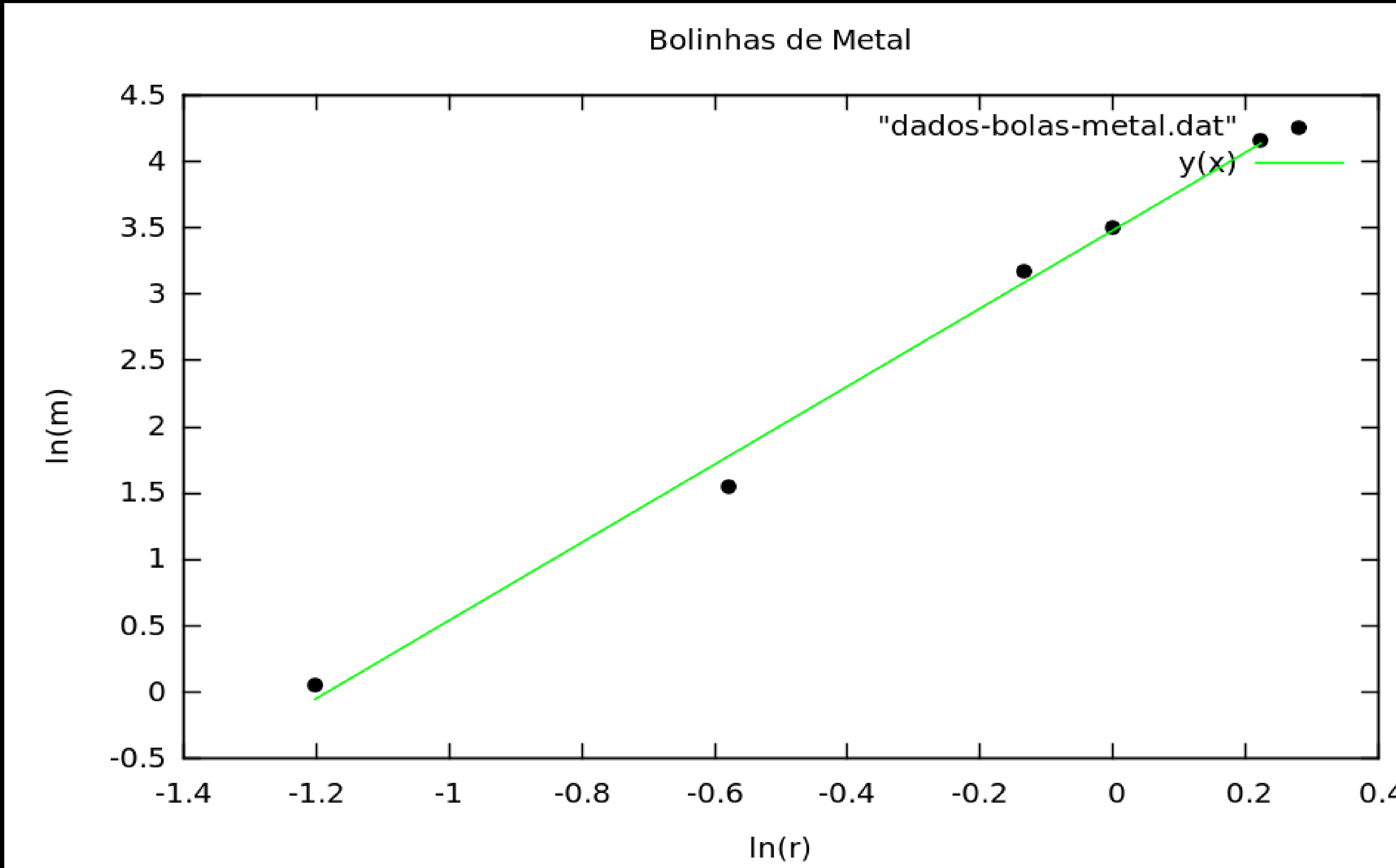
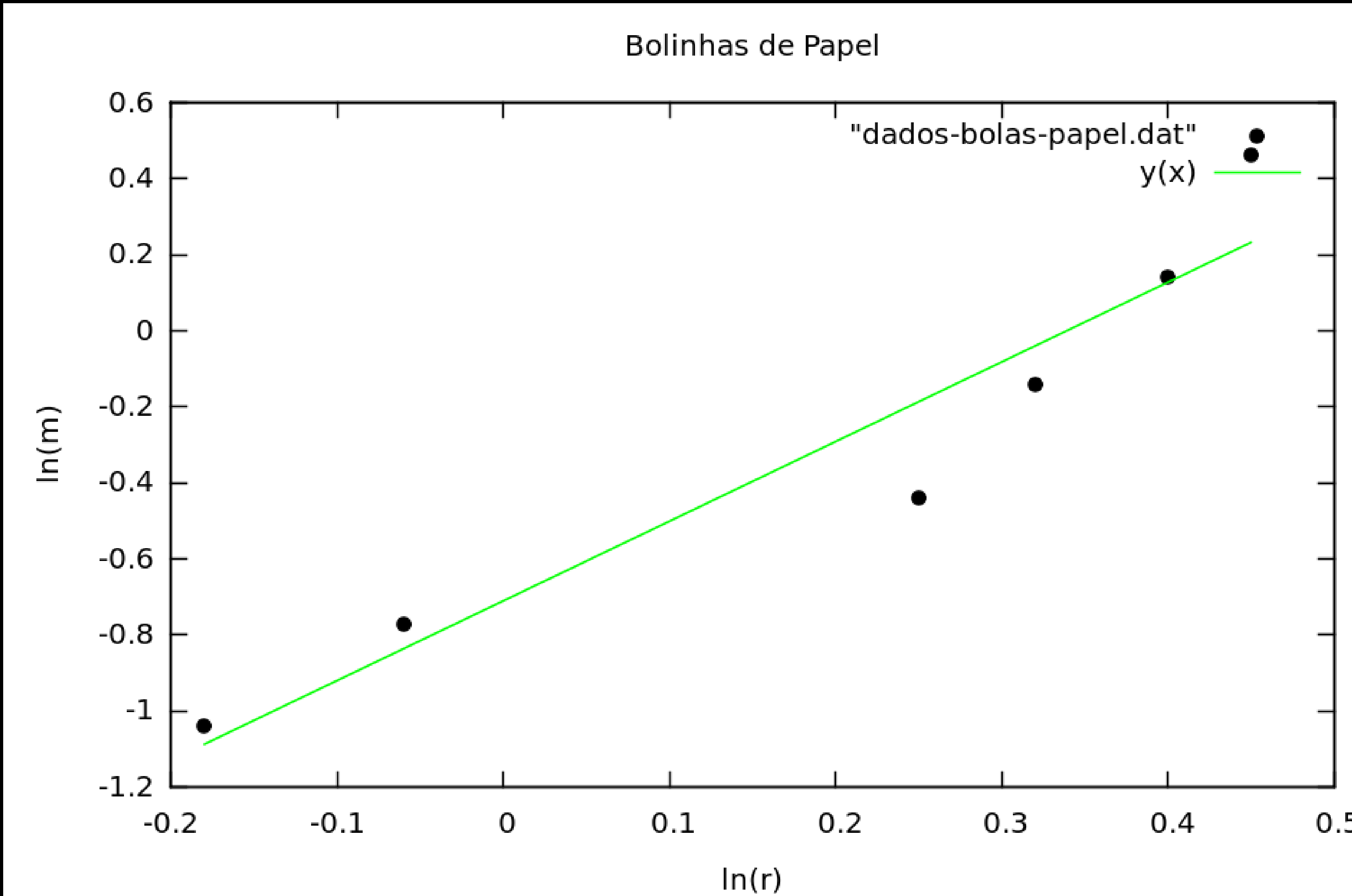


Fig. 3. The frequency distribution of the fractal dimension D in crumpled paper balls (surface density of the paper $\sigma \sim 80$ g/m^2) from data obtained by 89 students. $\bar{D} = 2.51 \pm 0.19$.




```
Final set of parameters            Asymptotic Standard Error
=======================            ==========================

d               = 2.93771          +/- 0.1347       (4.585%)
c               = 3.47989          +/- 0.08195      (2.355%)
```




```
Final set of parameters            Asymptotic Standard Error
=======================            ==========================

d               = 2.09346          +/- 0.3165       (15.12%)
c               = -0.710047        +/- 0.097        (13.66%)
```