Física e Biologia do Crescimento Populacional

Prof. Fabiano Ribeiro

DEX- UFLA

Prof. Dr.Fabiano Ribeiro - DEX- UFLA

Prof. Dr. Alexandre Martinez - USP

Dr. Brenno Cabella - Pós-Doc. USP

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Data collapse, scaling functions, and analytical solutions of generalized growth models

Brenno Caetano Troca Cabella* and Alexandre Souto Martinez†

Departamento de Física, Faculdade de Filosofia, Ciências e Letras de Ribeirão Preto, Universidade de São Paulo, 14040-901 Ribeirão Preto, São Paulo, Brazil and

National Institute of Science and Technology in Complex Systems, 14040-901 Ribeirão Preto, São Paulo, Brazil

Fabiano Ribeiro‡

Departamento de Ciências Exatas, Universidade Federal de Lavras, 37200-000 Lavras, Minas Gerais, Brazil

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We consider a nontrivial one-species population dynamics model with finite and infinite carrying capacities. Time-dependent intrinsic and extrinsic growth rates are considered in these models. Through the model *per capita* growth rate we obtain a heuristic general procedure to generate scaling functions to collapse data into a simple linear behavior even if an extrinsic growth rate is included. With this data collapse, all the models studied become independent from the parameters and initial condition. Analytical solutions are found when time-dependent coefficients are considered. These solutions allow us to perceive nontrivial transitions between species extinction and survival and to calculate the transition's critical exponents. Considering an extrinsic growth rate as a cancer treatment, we show that the relevant quantity depends not only on the intensity of the treatment, but also on when the cancerous cell growth is maximum.

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Effective carrying capacity and analytical solution of a particular case of the Richards-like two-species population dynamics model

Brenno Caetano Troca Cabella [a,*], Fabiano Ribeiro [b], Alexandre Souto Martinez [a,c]

[a] Faculdade de Filosofia, Ciências e Letras de Ribeirão Preto (FFCLRP), Universidade de São Paulo (USP), Avenida Bandeirantes, 3900, 14040-901, Ribeirão Preto, São Paulo, Brazil

[b] Departamento de Ciências Exatas (DEX), Universidade Federal de Lavras (UFLA), Caixa Postal 3037, 37200-000 Lavras, MG, Brazil

[c] Instituto Nacional de Ciência e Tecnologia em Sistemas Complexos, Brazil

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ABSTRACT

We consider a generalized two-species population dynamic model and analytically solve it for the amensalism and commensalism ecological interactions. These two-species models can be simplified to a one-species model with a time dependent extrinsic growth factor. With a one-species model with an effective carrying capacity one is able to retrieve the steady state solutions of the previous one-species model. The equivalence obtained between the effective carrying capacity and the extrinsic growth factor is complete only for a particular case, the Gompertz model. Here we unveil important aspects of sigmoid growth curves, which are relevant to growth processes and population dynamics.

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Full analytical solution and complete phase diagram analysis of the Verhulst-like two-species population dynamics model

Brenno Caetano Troca Cabella[1], **Fabiano Ribeiro**[2] and **Alexandre Souto Martinez**[1,3]

[1] Faculdade de Filosofia, Ciências e Letras de Ribeirão Preto (FFCLRP)
Universidade de São Paulo (USP)
Av. Bandeirantes, 3900
14040-901 Ribeirão Preto, SP, Brazil
[2] Departamento de Ciências Exatas (DEX),
Universidade Federal de Lavras (UFLA)
Caixa Postal 3037
37200-000 Lavras, MG, Brazil
[3] National Institute of Science and Technology in Complex Systems

E-mail: brenno@usp.br, flribeiro@dex.ufla.br, asmartinez@usp.br

Abstract. The two-species population dynamics model is the simplest paradigm of interspecies interaction. Here, we include intraspecific competition to the Lotka-Volterra model and solve it analytically. Despite being simple and thoroughly studied, this model presents a very rich behavior and some characteristics not so well explored, which are unveiled. The forbidden region in the mutualism regime and the dependence on initial conditions in the competition regime are some examples of these characteristics. From the stability of the steady state solutions, three phases are obtained: (i) extinction of one species (Gause transition), (ii) their coexistence and (iii) a forbidden region. Full analytical solutions have been obtained for the considered ecological regimes. The time transient allows one to defined time scales for the system evolution, which can be relevant for the study of tumor growth by theoretical or computer simulation models.

Generalized competitive Lotka-Volterra model

Fabiano Ribeiro*
Departamento de Ciências Exatas (DEX),
Universidade Federal de Lavras (UFLA)
Caixa Postal 3037
37200-000 Lavras, Minas Gerais, Brazil

Brenno Caetano Troca Cabella† and Alexandre Souto Martinez‡
Faculdade de Filosofia,
Ciências e Letras de Ribeirão Preto (FFCLRP),
Universidade de São Paulo (USP),
Avenida dos Bandeirantes, 3900, 14.040-901,
Ribeirão Preto, SP, Brazil and
Instituto Nacional de Ciência e Tecnologia em Sistemas Complexos
(Dated: November 26, 2012)

The two-species population dynamics model is the simplest paradigm of inter- and intra-species interaction. Here, we present a generalized competitive Lotka-Volterra model, which retrieves as particular cases, some well known two-population dynamic models. We identify the model generalization parameter with the environmental fractal dimension and the individuals interaction range. The species interaction parameters are not restricted, consequently they can represent different ecological regimes. The asymptotic solutions stability analysis leads to represent the ecological regimes as a phase diagram in the parameter space. In this diagram we call attention to: a forbidden region in the mutualism regime and to the region with dependence on initial conditions, in the competition regime. Also, we have identified predation and competition to be weak, if there are species coexistence or strong if there are species extinction. Moreover, contrasting to known two-species models,

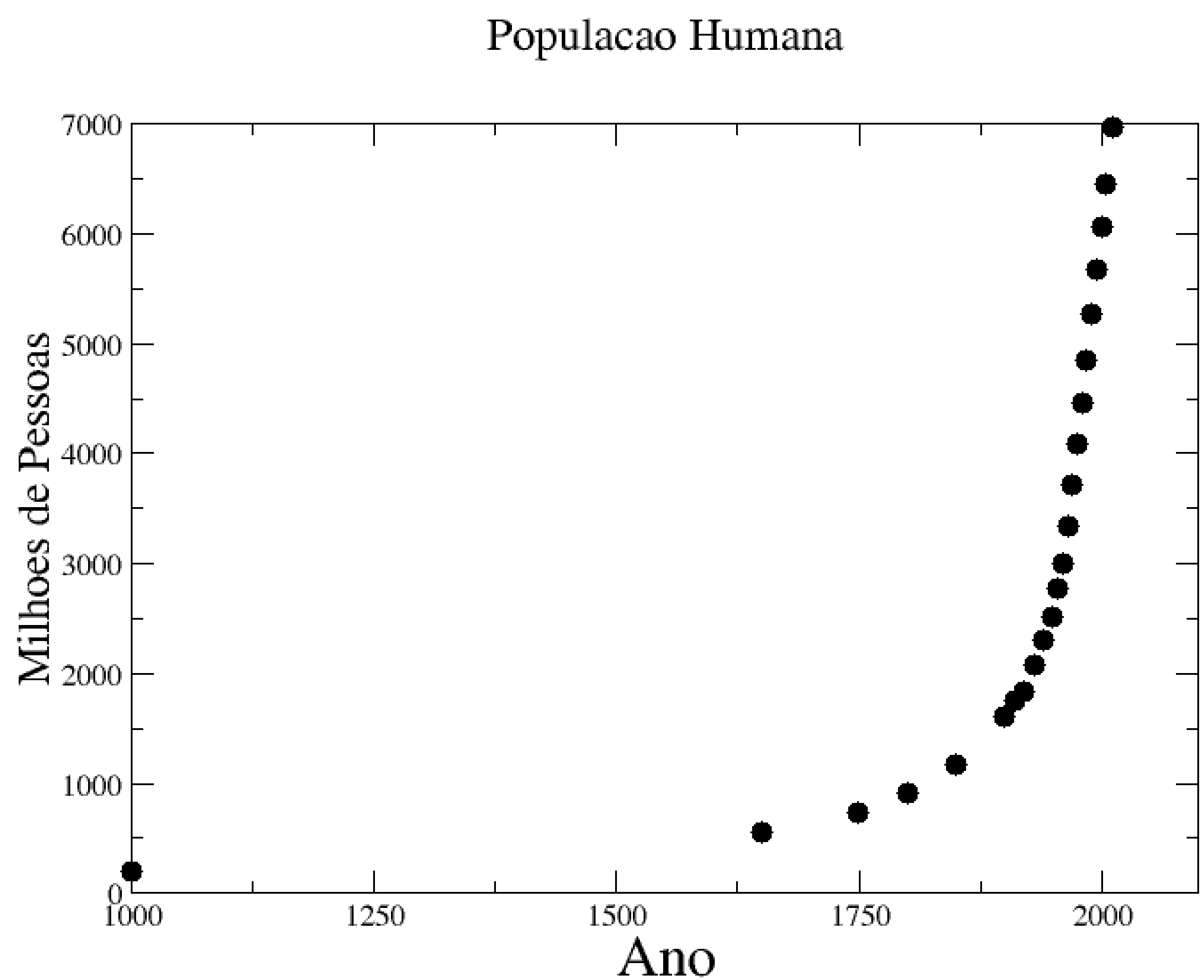

DADOS
Lucro

World human population.

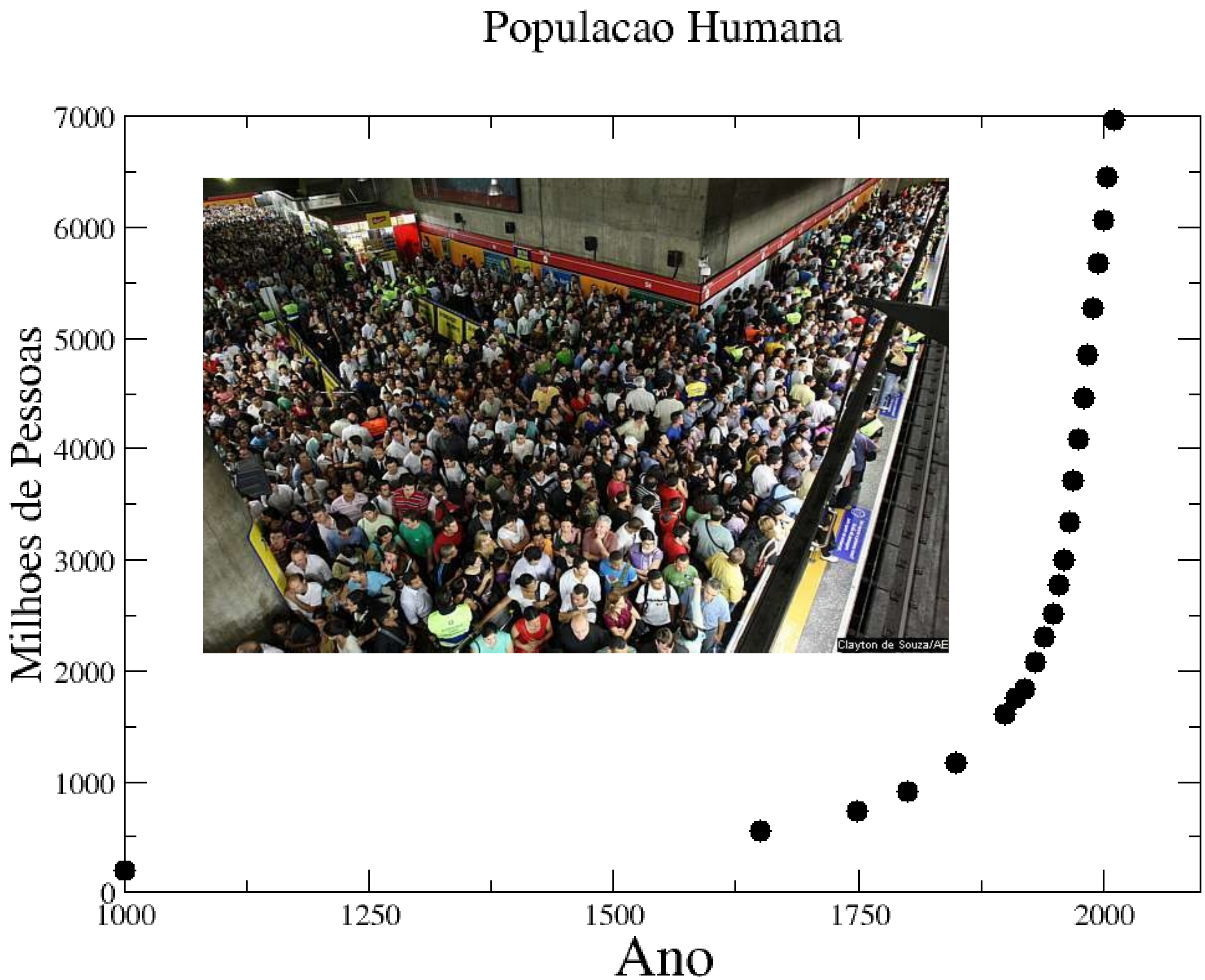
Year	Population (millions)	Year	Population (millions)
1000	200	1955	2780
1650	545	1960	3005
1750	728	1965	3345
1800	906	1970	3707
1850	1171	1975	4086
1900	1608	1980	4454
1910	1750	1985	4850
1920	1834	1990	5263
1930	2070	1995	5674
1940	2295	2000	6070
1950	2517	2005	6453

18/06/2013: 7.092.566.780 Habitantes

Fonte: Fundo de População das Nações unidas (UNFPA/ONU)



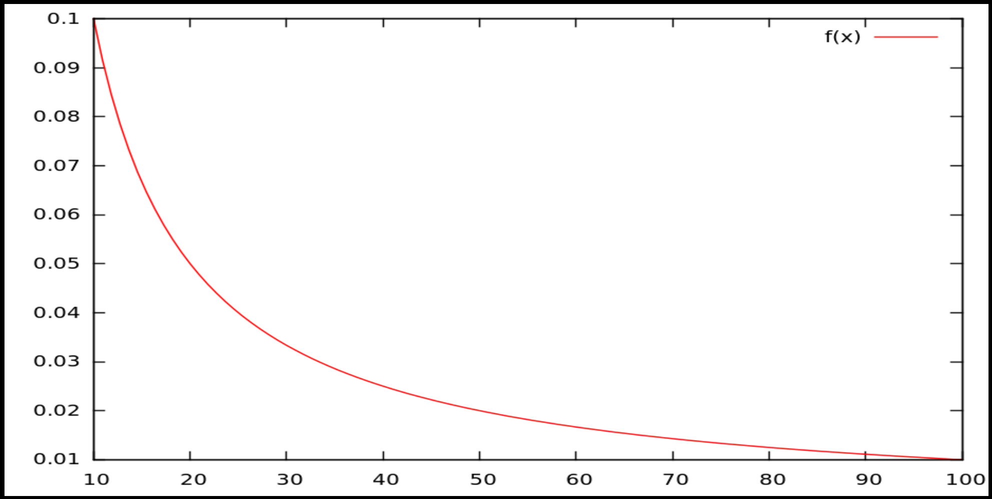
Populacao Humana
Milhoes de Pessoas
0
1000
2000
3000
4000
5000
6000
7000
1000
1250
1500
1750
2000
Ano



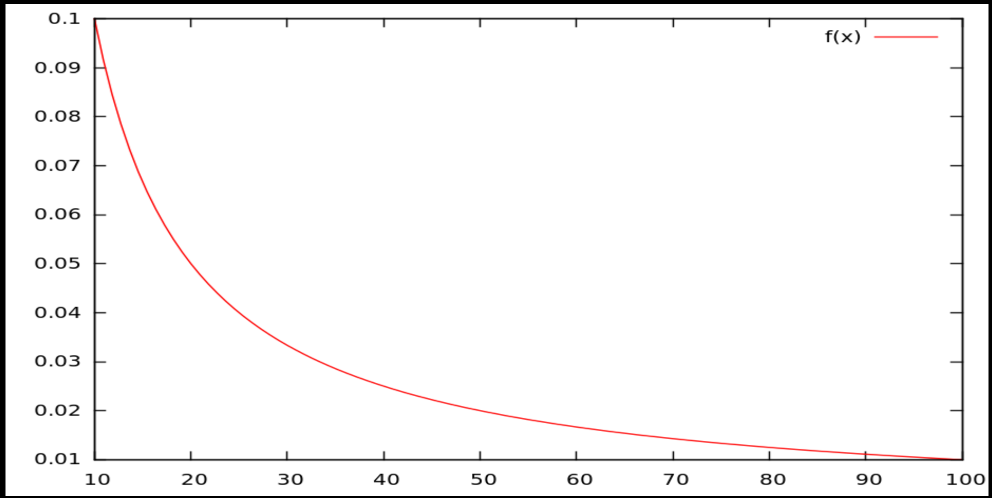
Populacao Humana
Milhoes de Pessoas
0
1000
2000
3000
4000
5000
6000
7000
1000
1250
1500
1750
2000
Ano
Clayton de Souza/AE

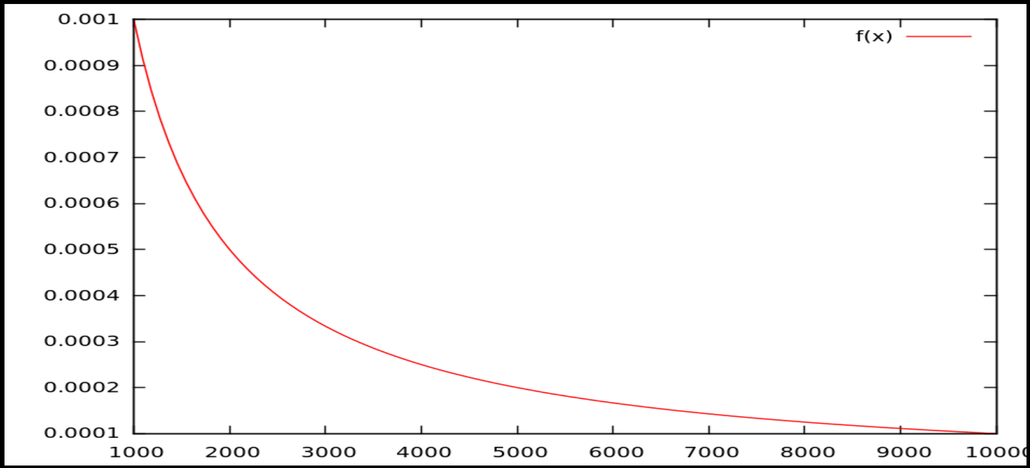

Blue whale Balaenoptera musculus

Busca de uma
Lei do Crescimento

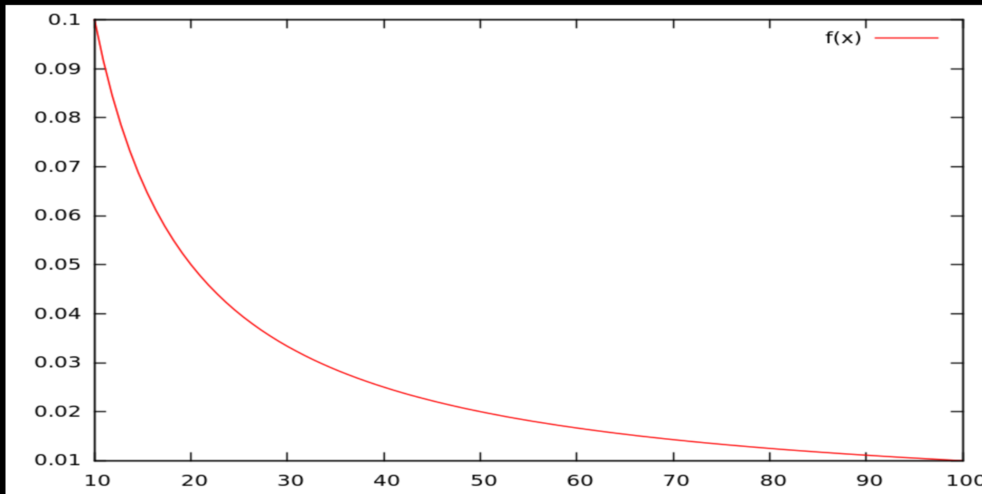

0.1
0.09
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
10
20
30
40
50
60
70
80
90
100
f(x)

$$y(x)=x^{-1}$$


0.1
0.09
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
10 20 30 40 50 60 70 80 90 100
f(x)


0.001
0.0009
0.0008
0.0007
0.0006
0.0005
0.0004
0.0003
0.0002
0.0001
1000 2000 3000 4000 5000 6000 7000 8000 9000 10000
f(x)

$$y(x)=x^{-1}$$


f(x)
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0.09
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
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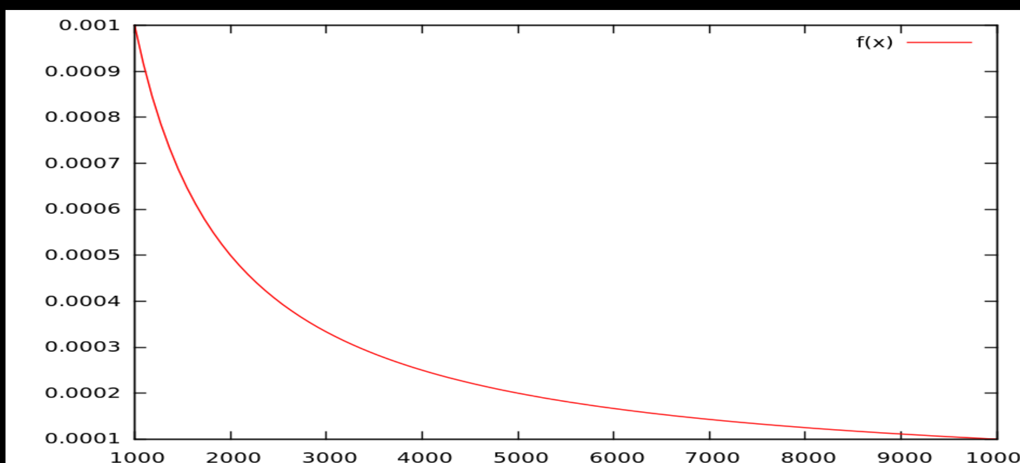
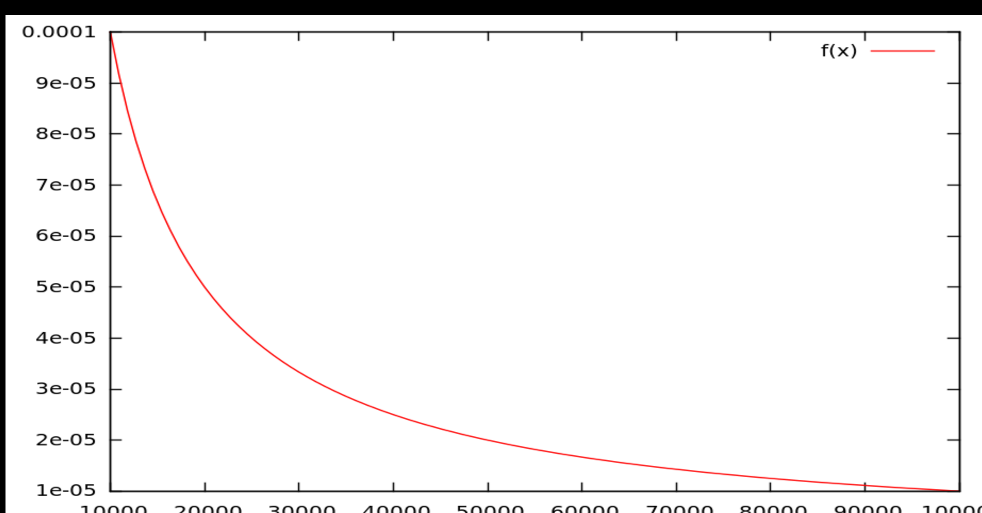
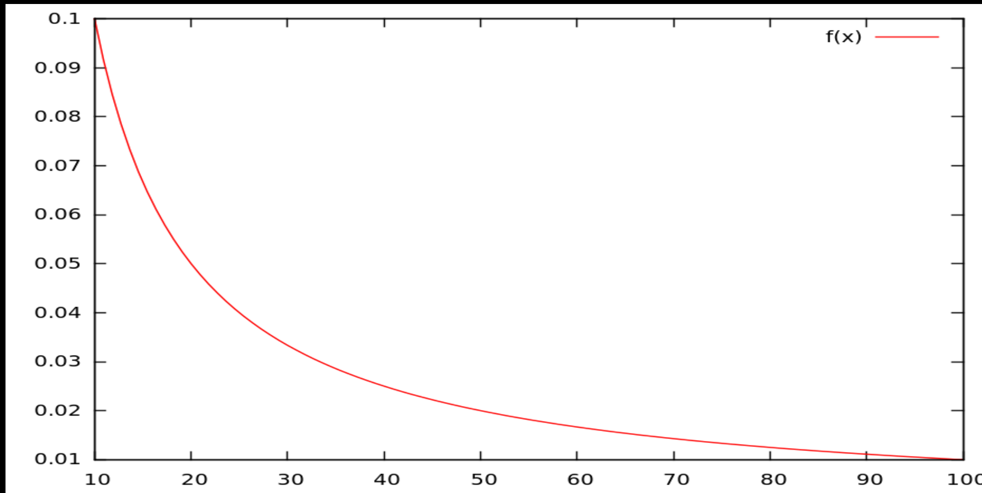
30
40
50
60
70
80
90
100

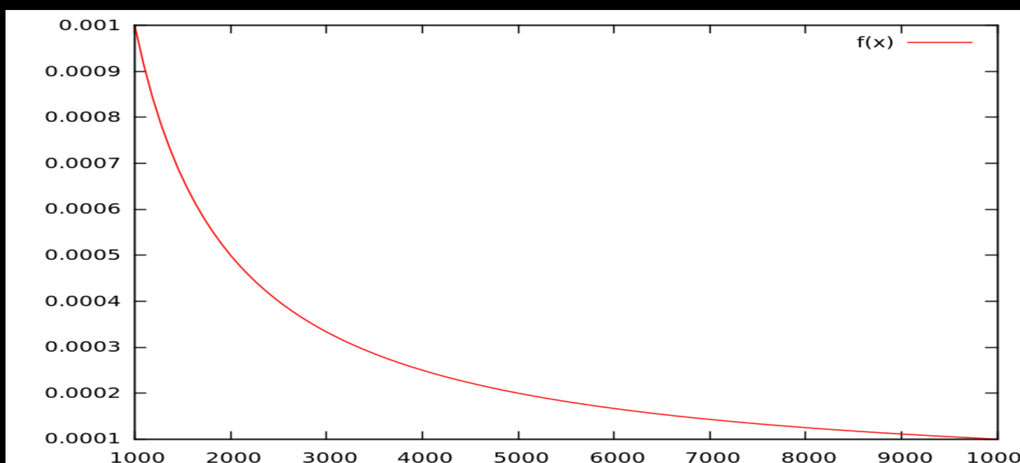

f(x)
0.001
0.0009
0.0008
0.0007
0.0006
0.0005
0.0004
0.0003
0.0002
0.0001
1000
2000
3000
4000
5000
6000
7000
8000
9000
10000

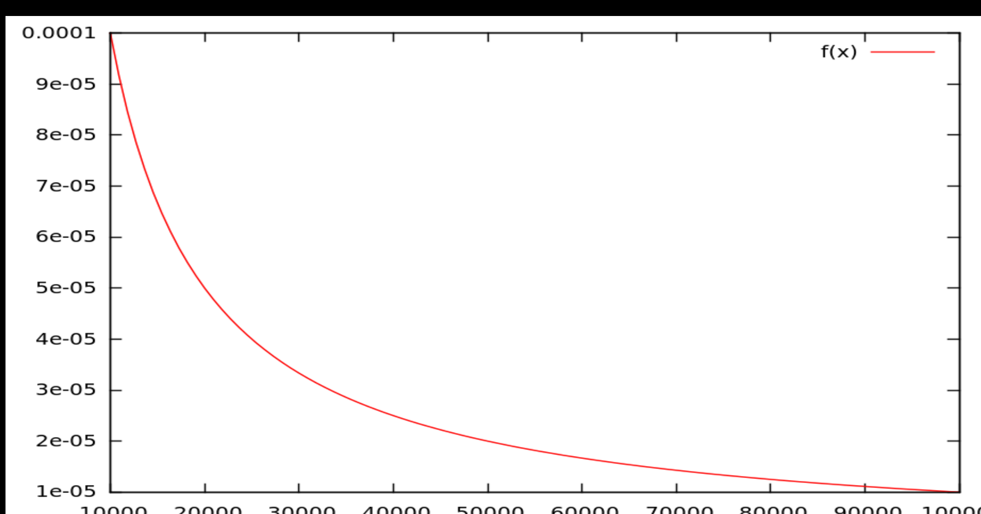

f(x)
0.0001
9e-05
8e-05
7e-05
6e-05
5e-05
4e-05
3e-05
2e-05
1e-05
10000
20000
30000
40000
50000
60000
70000
80000
90000
10000

$$y(x)=x^{-1}$$

$$y(x)=x^{-1}$$


f(x)
0.1
0.09
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
10 20 30 40 50 60 70 80 90 100


f(x)
0.001
0.0009
0.0008
0.0007
0.0006
0.0005
0.0004
0.0003
0.0002
0.0001
1000 2000 3000 4000 5000 6000 7000 8000 9000


f(x)
0.0001
9e-05
8e-05
7e-05
6e-05
5e-05
4e-05
3e-05
2e-05
1e-05
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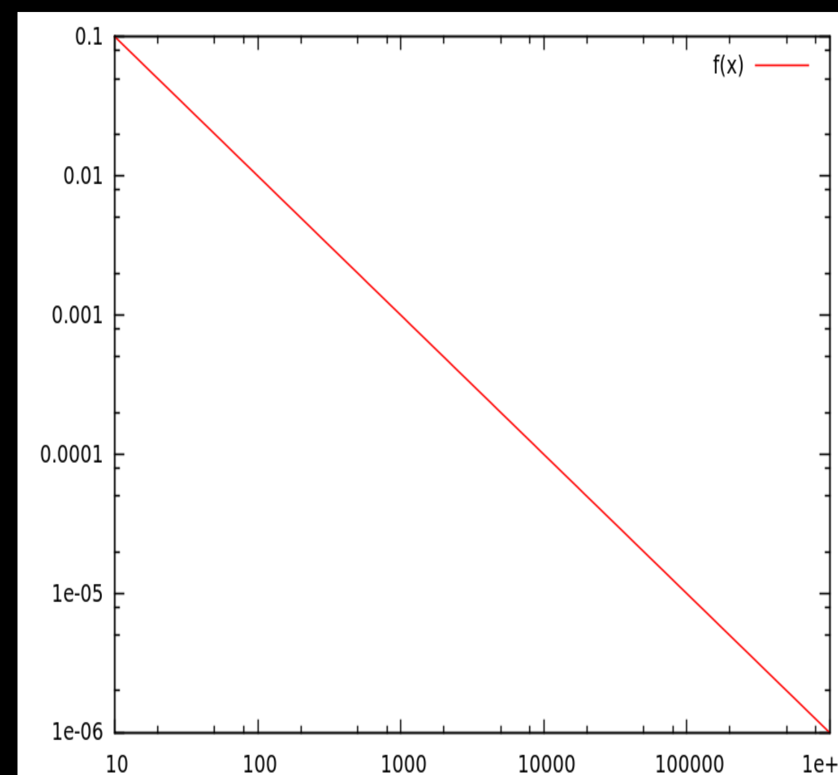
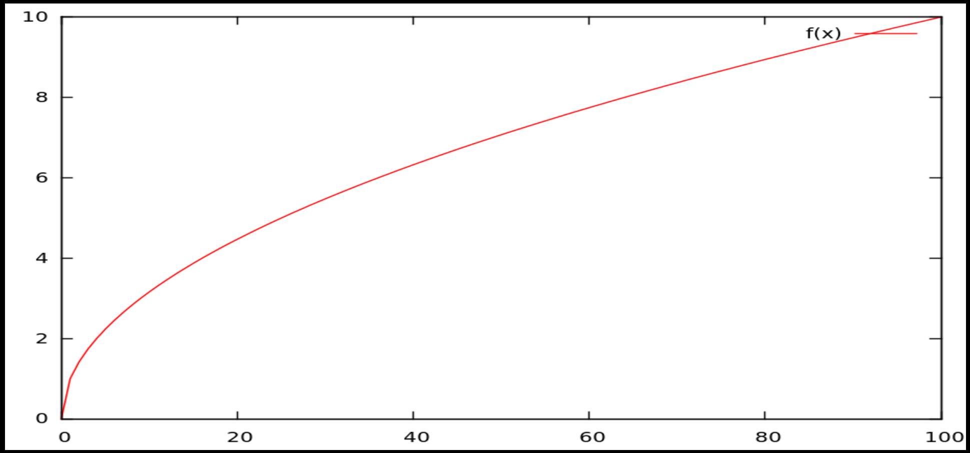
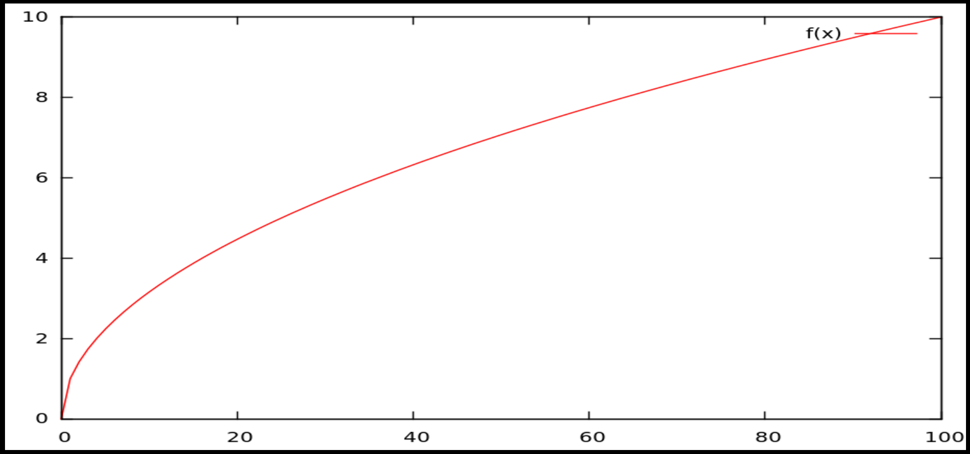
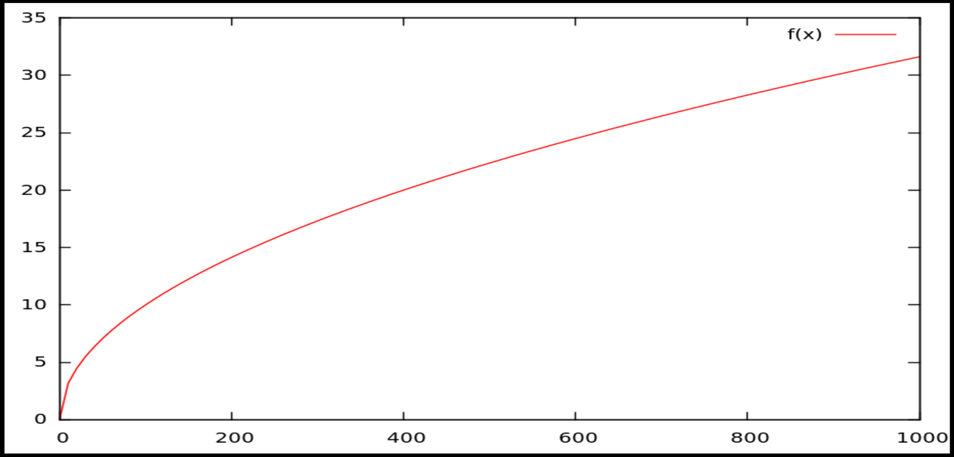

f(x)
0.1
0.01
0.001
0.0001
1e-05
1e-06
10 100 1000 10000 100000

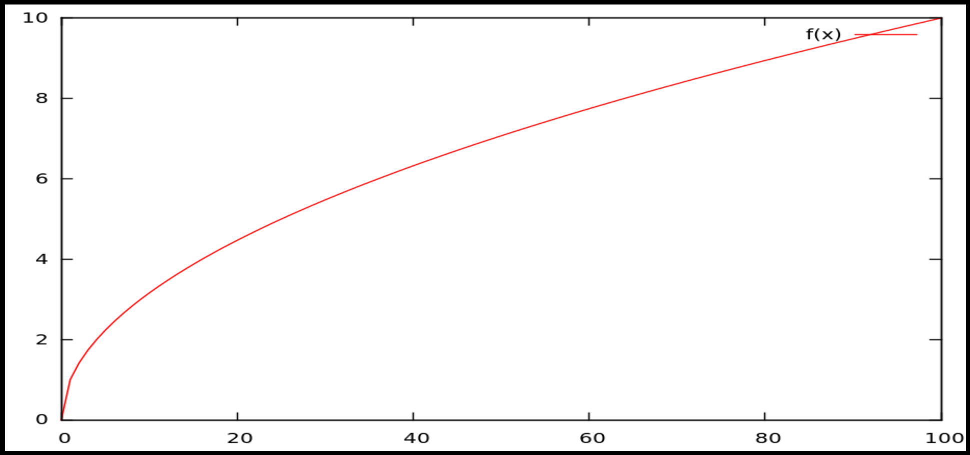


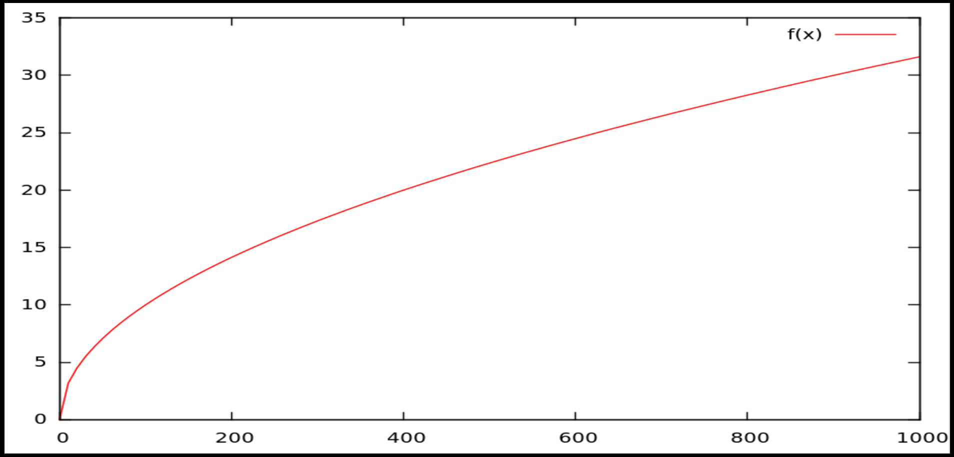
$$y(x)=x^{1/2}$$

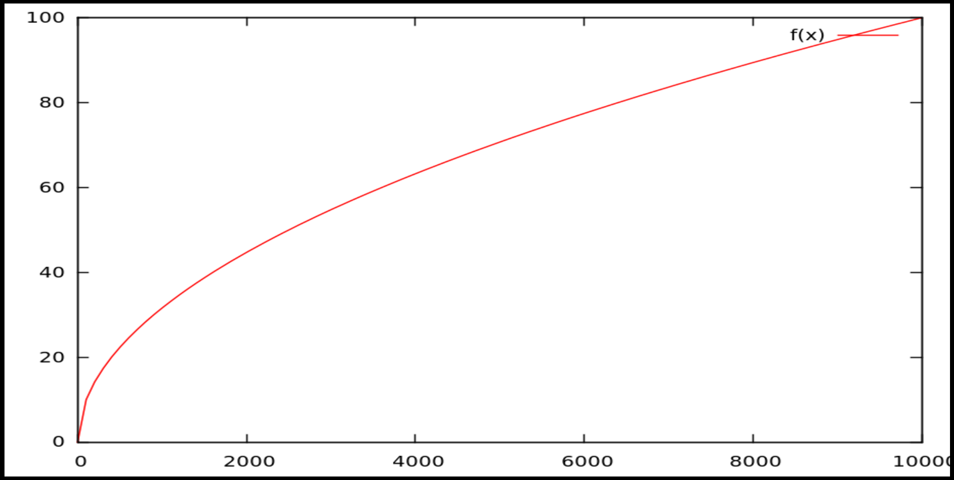

f(x)
0
2
4
6
8
10
0
20
40
60
80
100


f(x)
0
5
10
15
20
25
30
35
0
200
400
600
800
1000

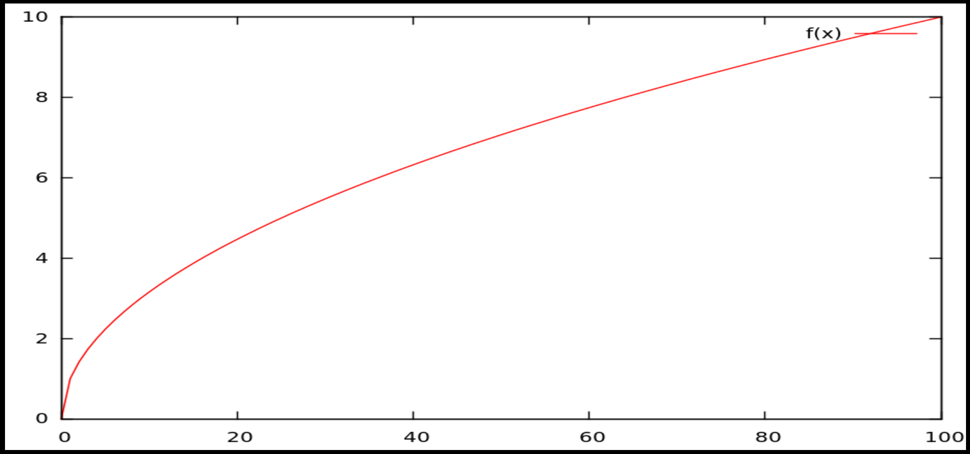
$$y(x)=x^{1/2}$$


f(x)
0
2
4
6
8
10
0
20
40
60
80
100


f(x)
0
5
10
15
20
25
30
35
0
200
400
600
800
1000

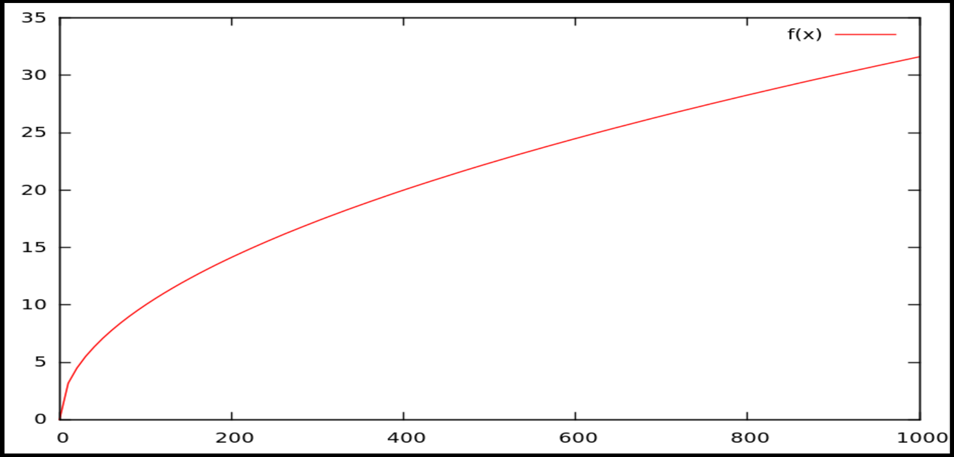

f(x)
0
20
40
60
80
100
0
2000
4000
6000
8000
10000

$$y(x)=x^{1/2}$$

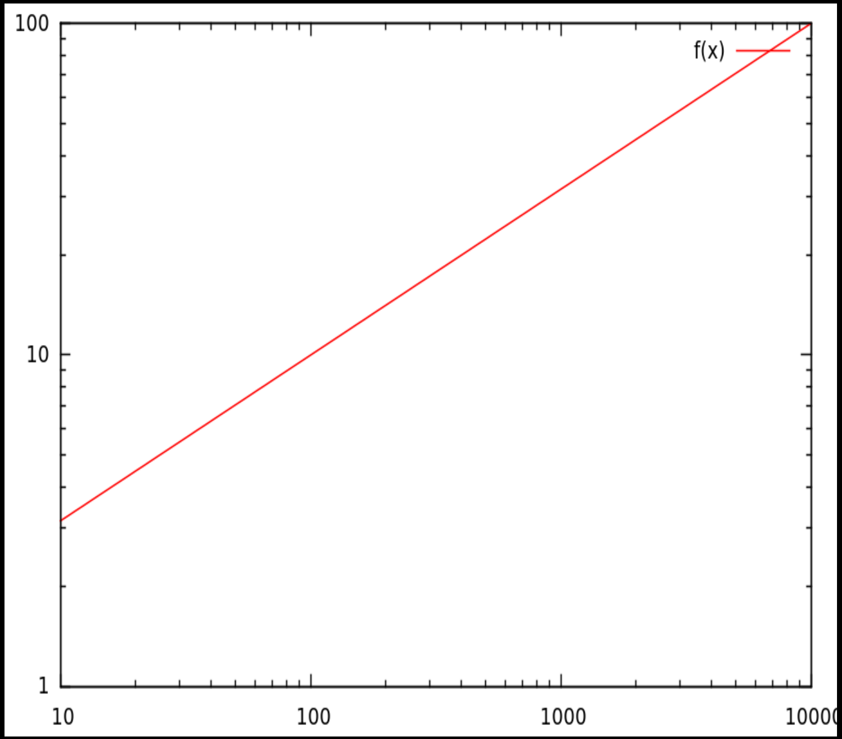


f(x)
0
2
4
6
8
10
0
20
40
60
80
100

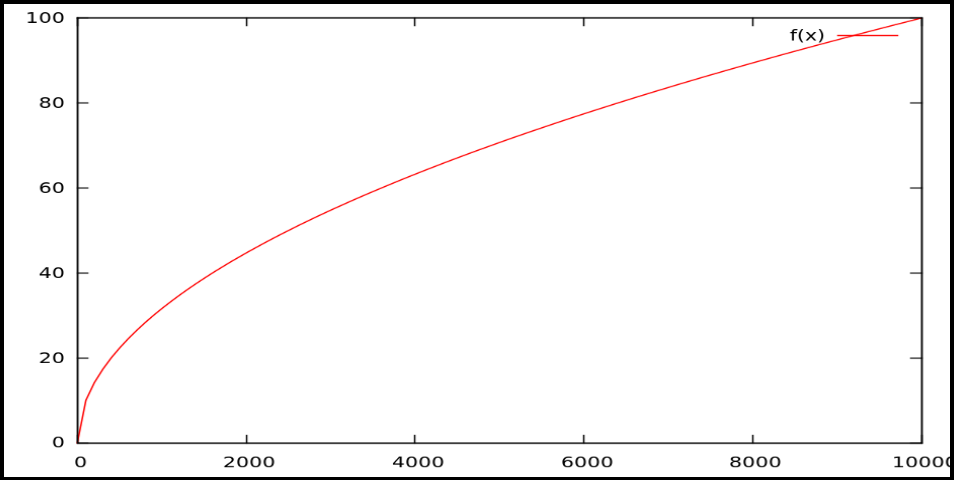
$$y(x)=x^{1/2}$$



f(x)
0
5
10
15
20
25
30
35
0
200
400
600
800
1000



f(x)
1
10
100
10
100
1000



f(x)
0
20
40
60
80
100
0
2000
4000
6000
8000

Lei de Potência:

$$y(x) = a\, x^{\beta}$$

For example, James Joyce's novel *Ulysses* contains 260,430 words. If words such as *give*, *gives*, *gave*, *given*, *giving*, *giver*, and *gift* are considered to be different, there are, in *Ulysses*, 29,899 different words. Zipf data, taken from *Human Behavior and the Principle of Least Effort*, p. 24, are reproduced in the table below.

r	f	r	f	r	f
10	2653	200	133	3000	8
20	1311	300	84	4000	6
30	926	400	62	5000	5
40	717	500	50	10,000	2
50	556	1000	20	20,000	1
100	265	2000	12	29,899	1

A log-log plot of the word frequency f as a function of the word rank r is shown in Figure 8.2 with a straight line representing the least-squares fit. The slope of this line is equal to 1.02.

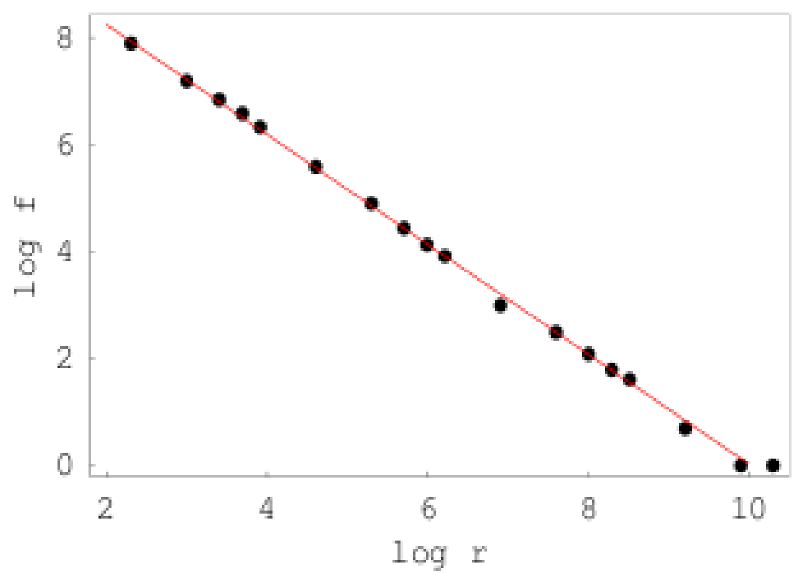
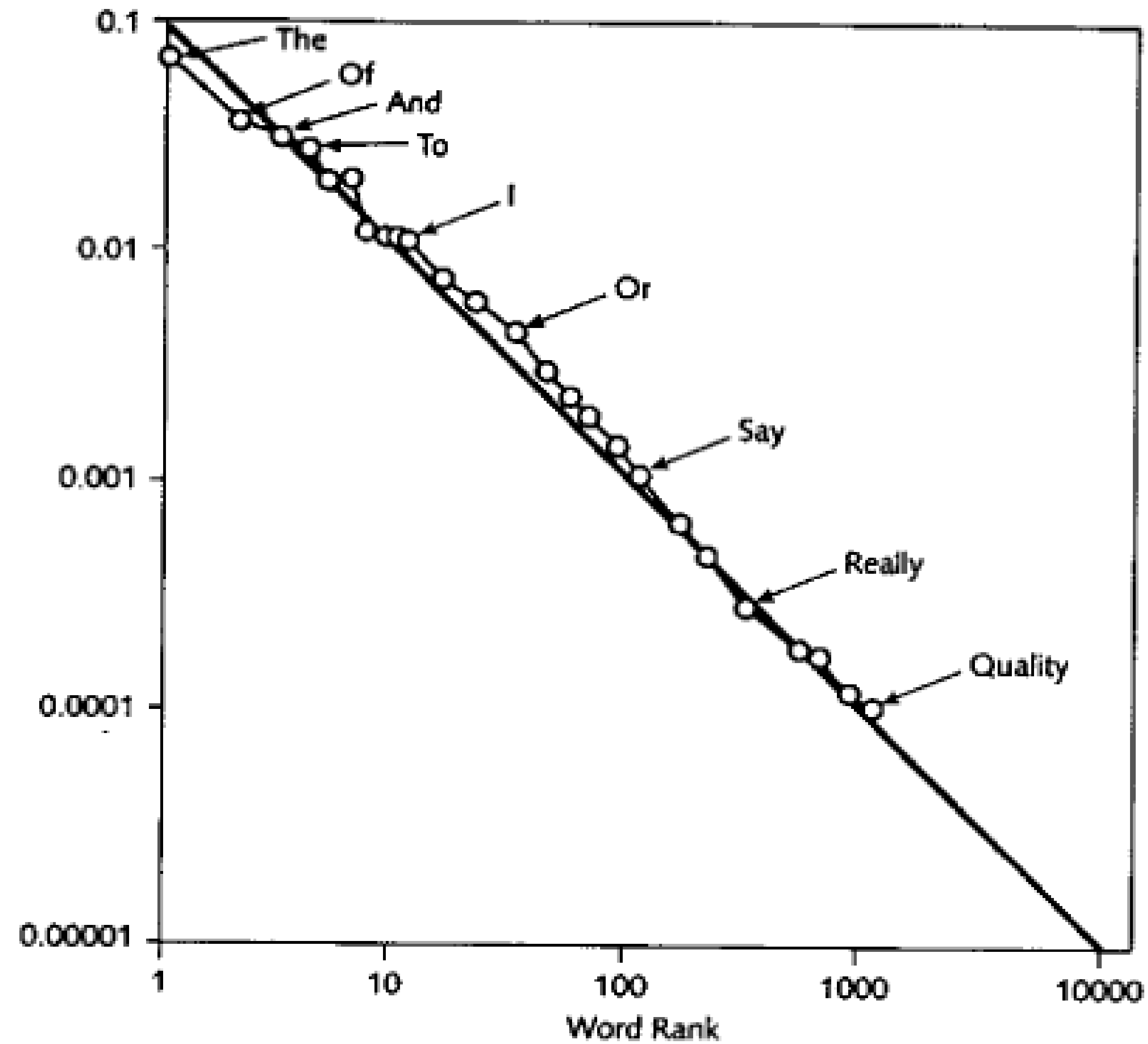
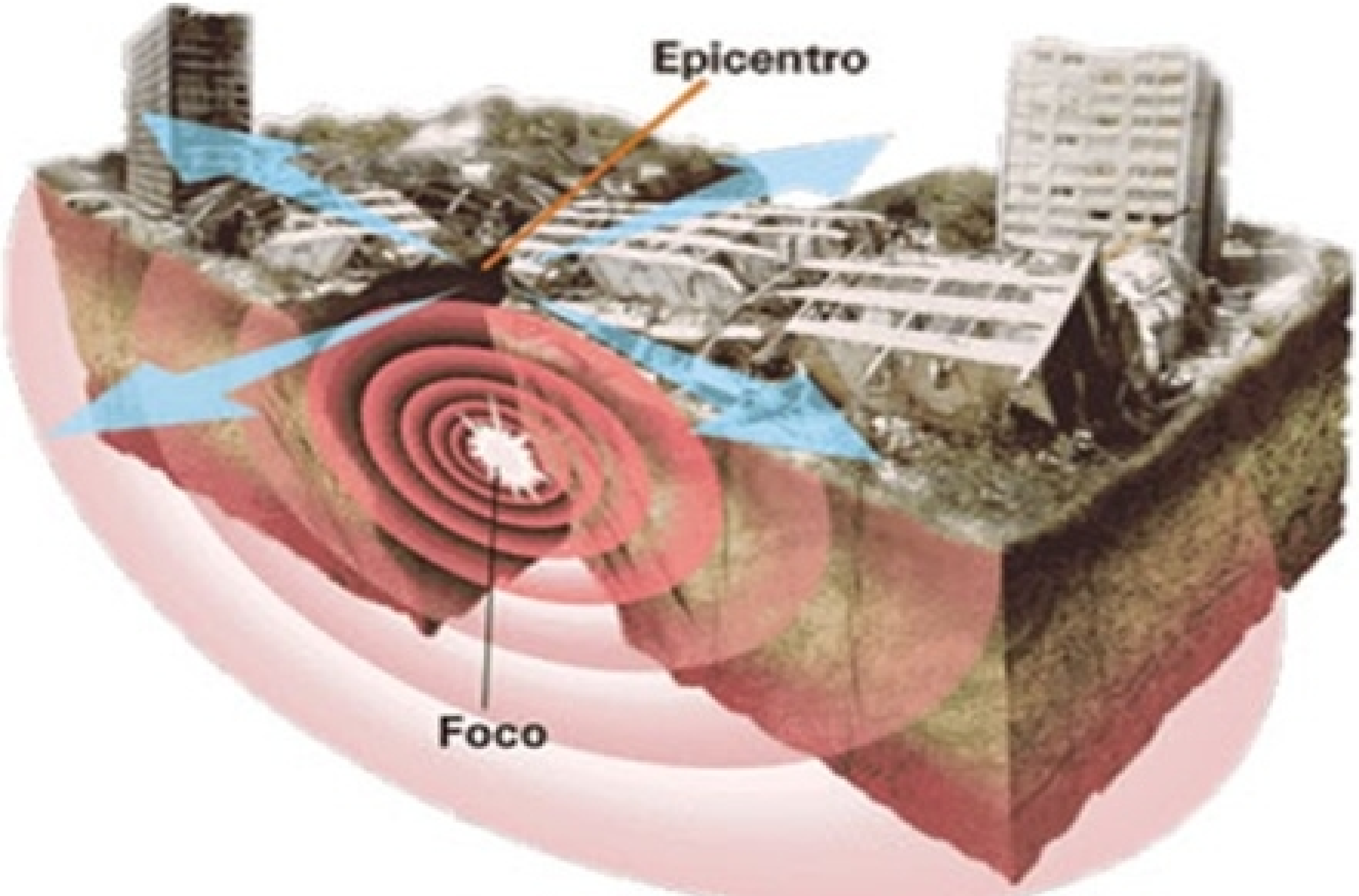


Fig. 8.2. *Log-log plot of word frequency f as a function of word rank r (dots) and the least-squares fit (straight line). Zipf data taken from Human Behavior and the Principle of Least Effort are reproduced in the table above.*

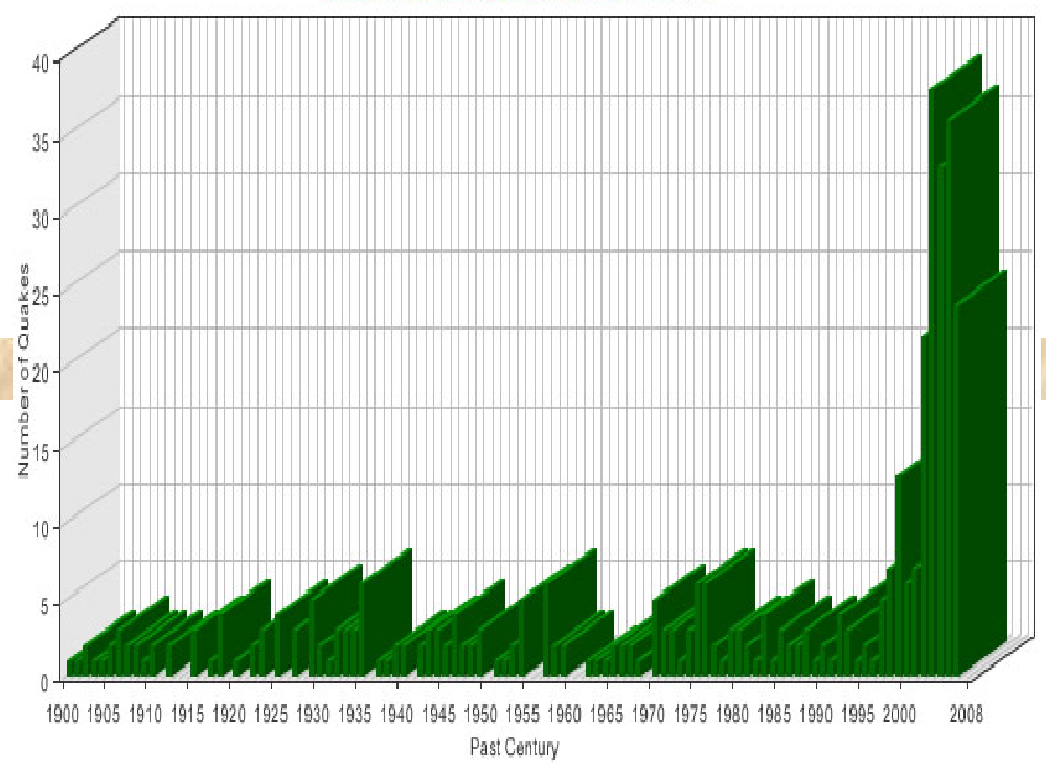


Zipf's Law for the English language.

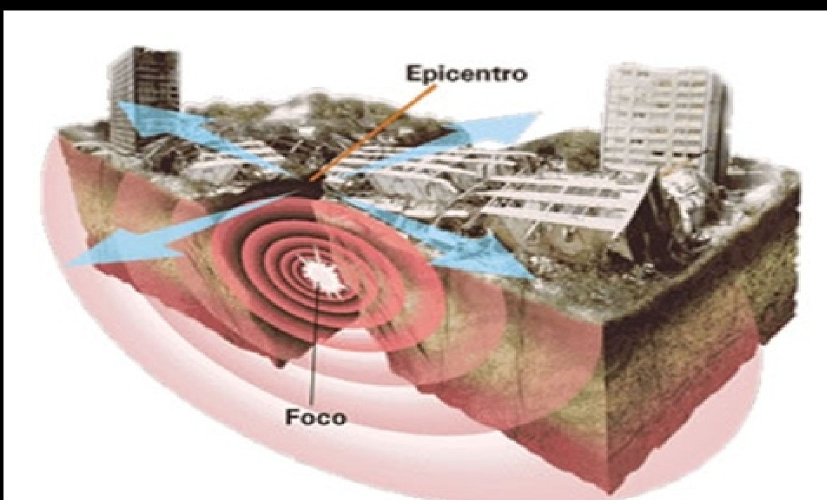
Figure 8. Continued (b) Ranking of words in the English language. The curve shows how many words appear with more than a given frequency.



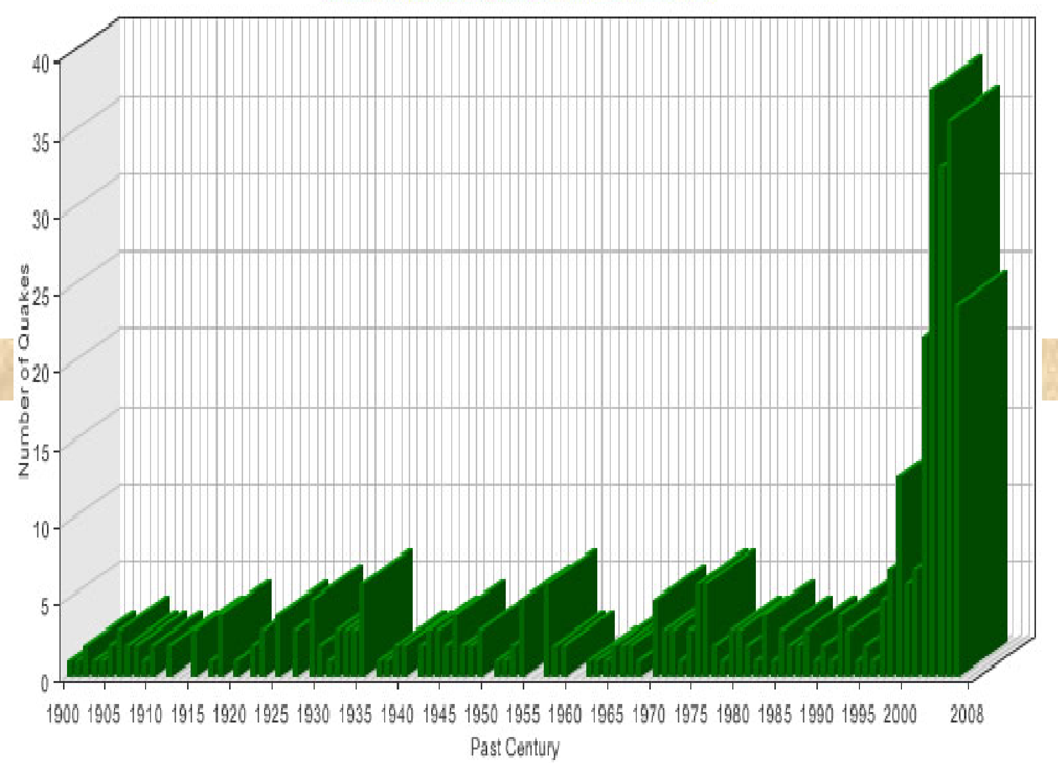
Demonstração do hipocentro (foco) e do epicentro de um terremoto



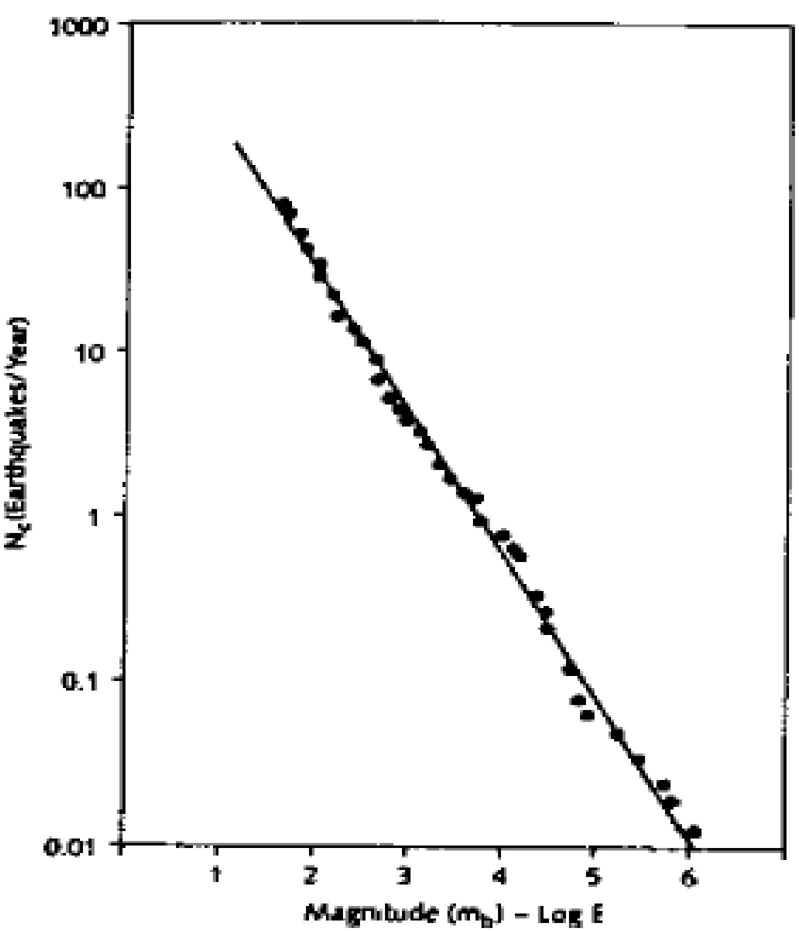
Incidência de terremotos de 1900 a 2008

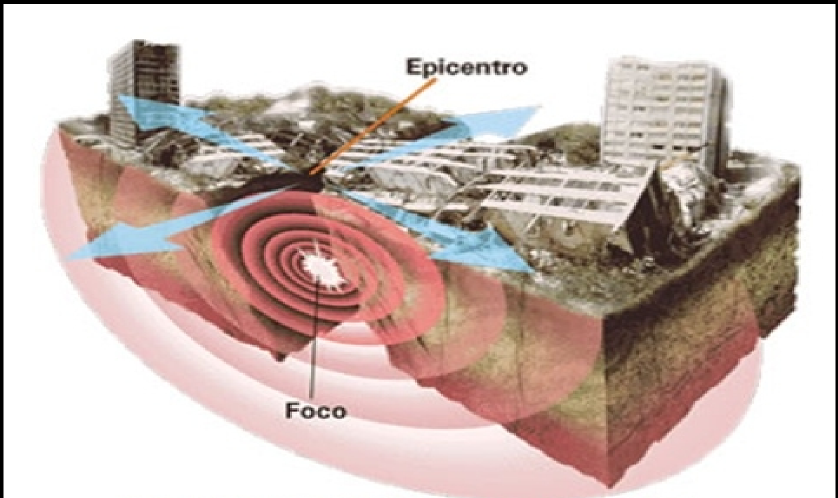


Demonstração do hipocentro (foco) e do epicentro de um terremoto

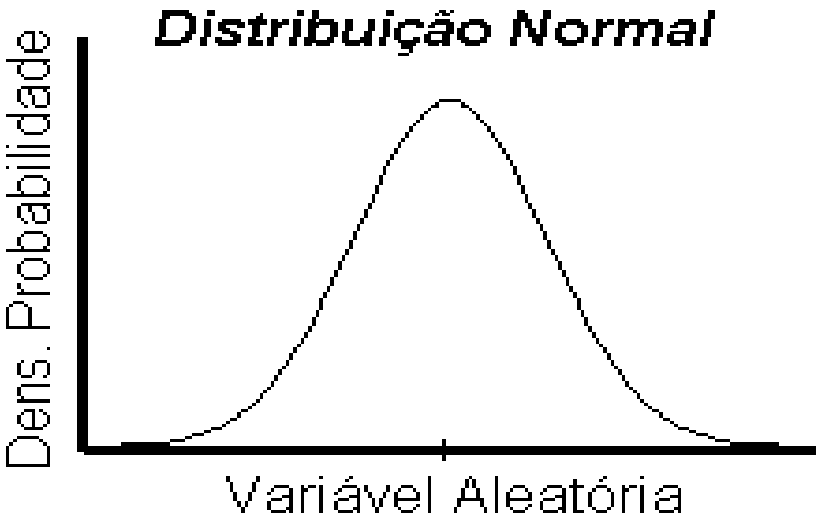


Incidência de terremotos de 1900 a 2008

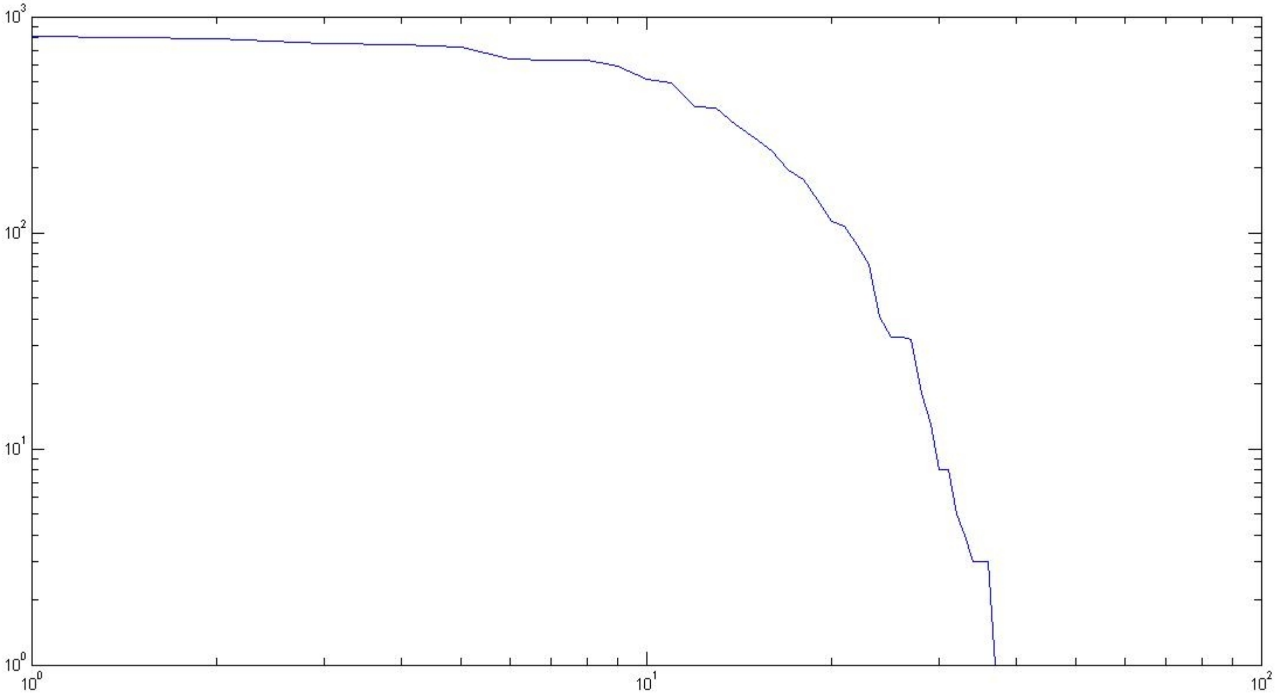




Demonstração do hipocentro (foco) e do epicentro de um terremoto



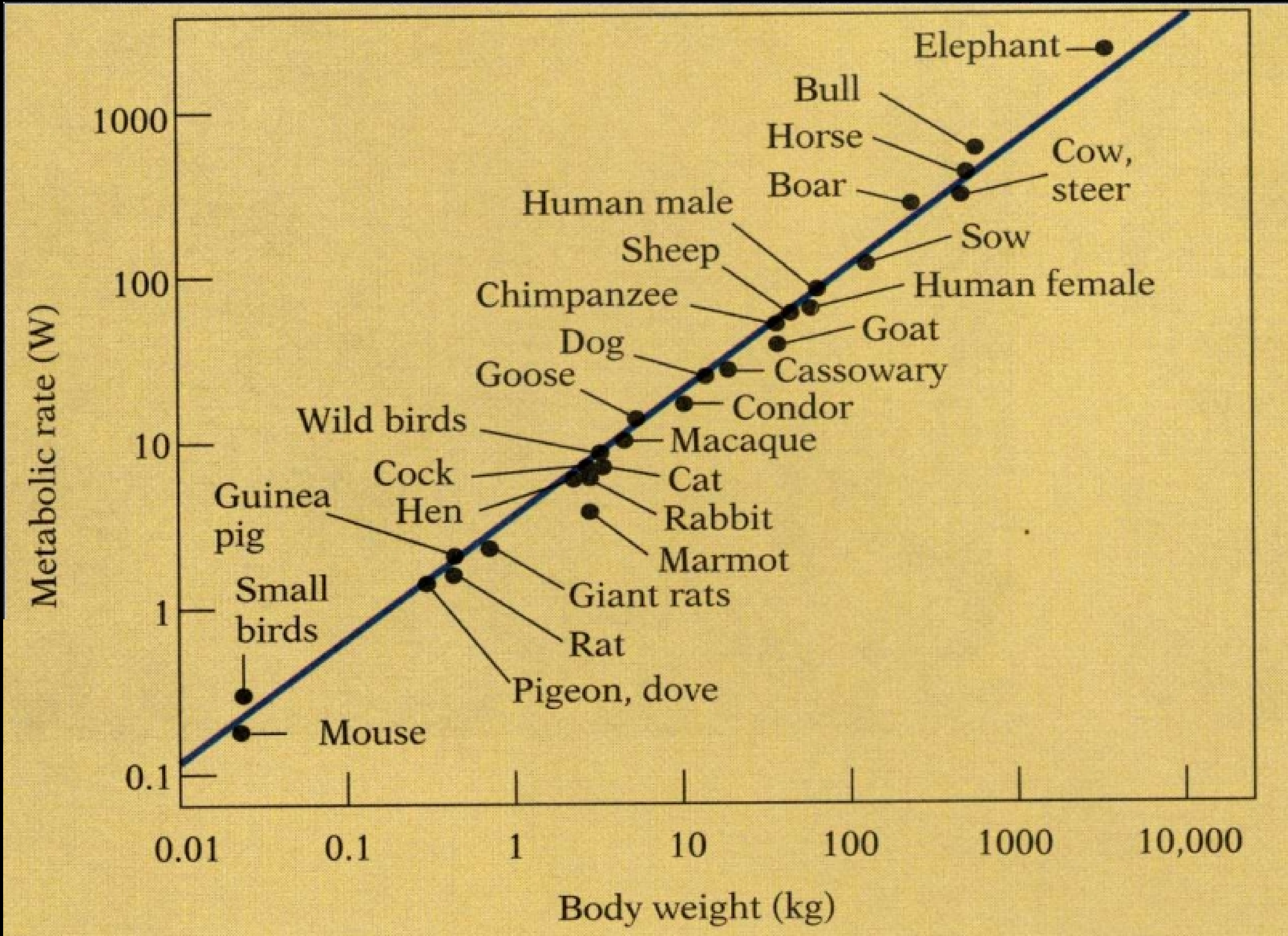
Frequencia



Ranqueamento

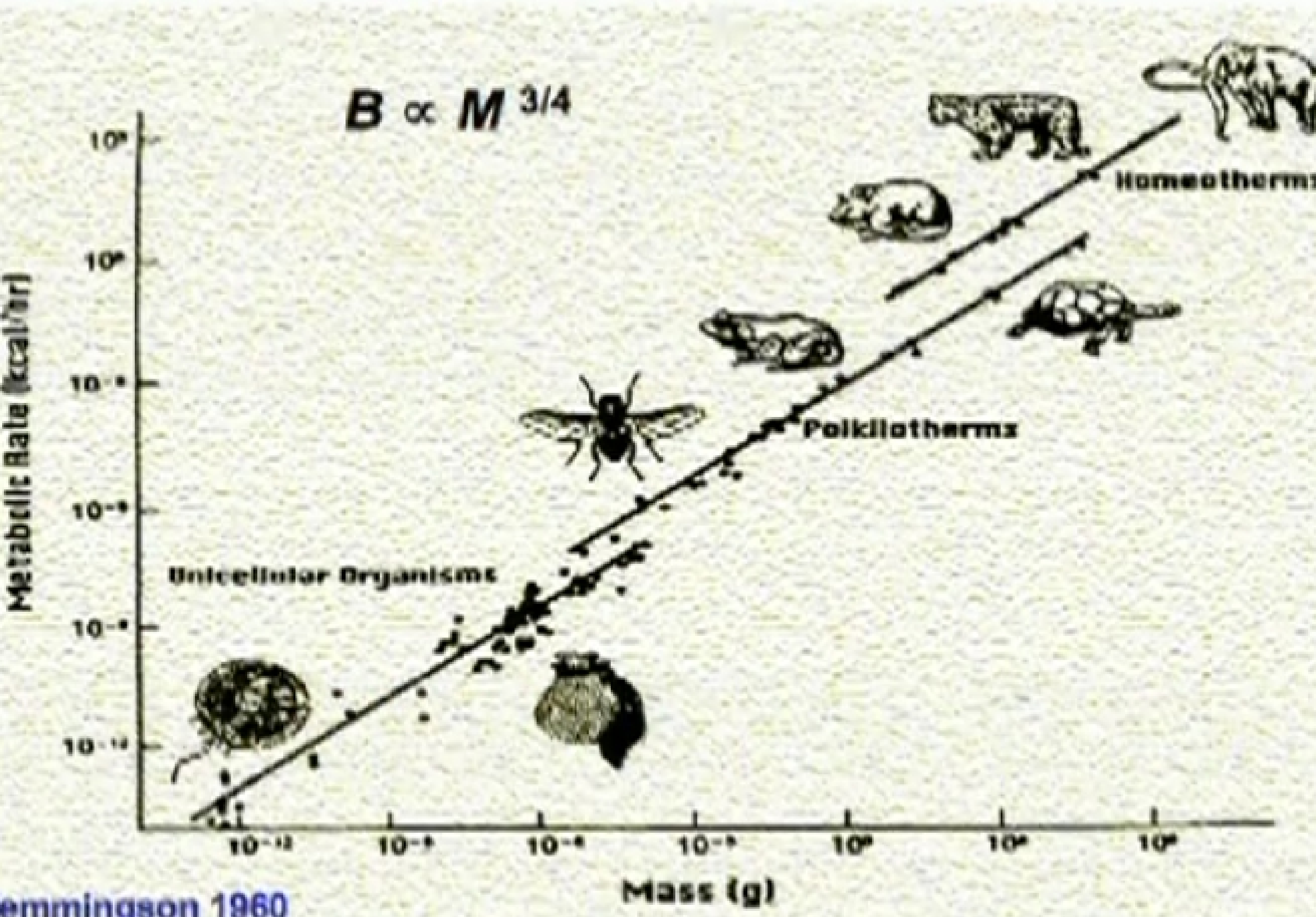
Leis de Escala em Biologia?

Mammals vary in size by 8 orders of magnitude

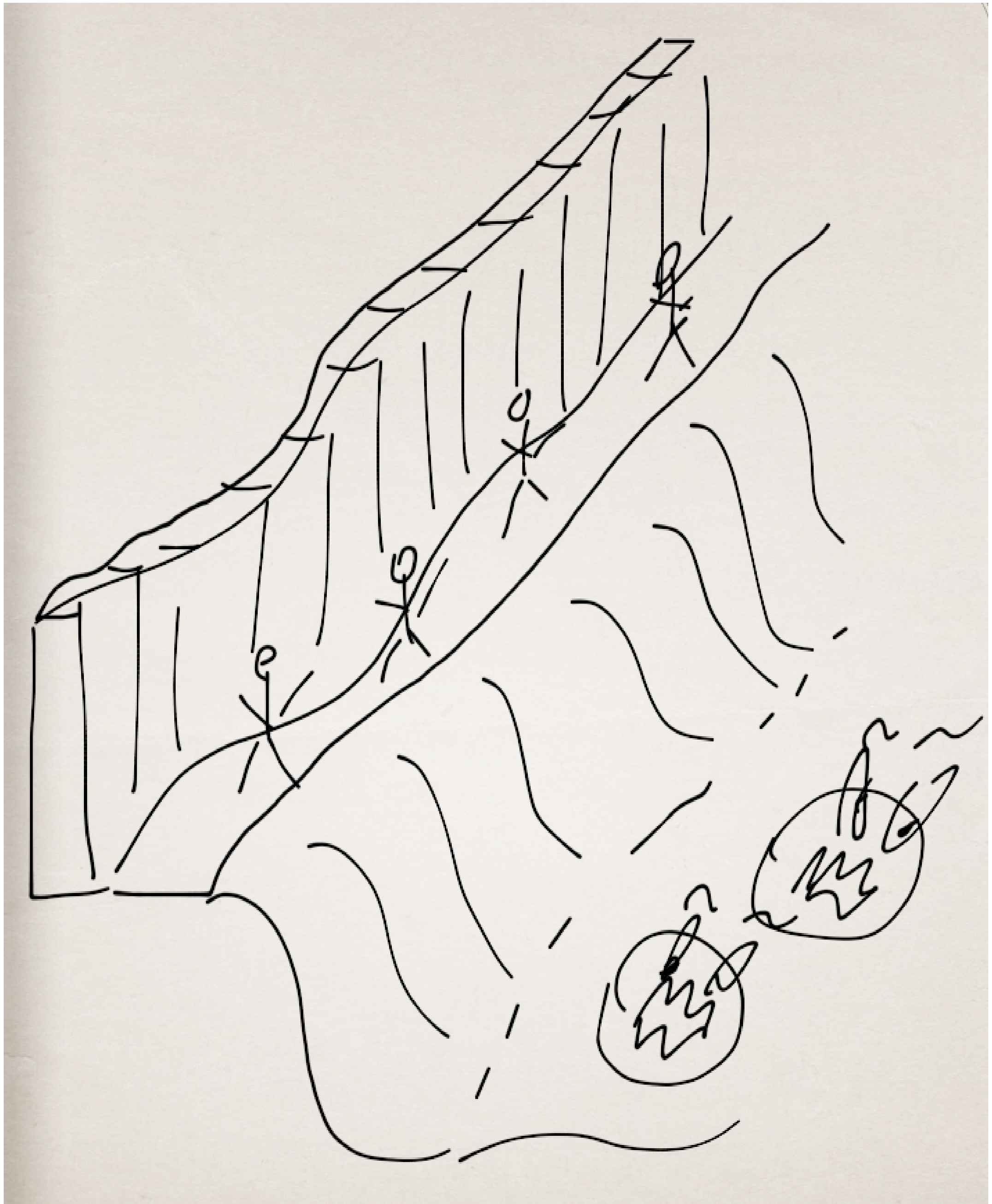


Elephant
Bull
Horse
Cow, steer
Boar
Human male
Sow
Sheep
Human female
Chimpanzee
Goat
Dog
Cassowary
Goose
Condor
Wild birds
Macaque
Cock
Cat
Hen
Rabbit
Guinea pig
Marmot
Giant rats
Small birds
Rat
Pigeon, dove
Mouse
Metabolic rate (W)
1000
100
10
1
0.1
0.01
0.1
1
10
100
1000
10,000
Body weight (kg)

Whole-organism metabolic rate (*B*) scales as the 3/4 power of body mass (*M*)



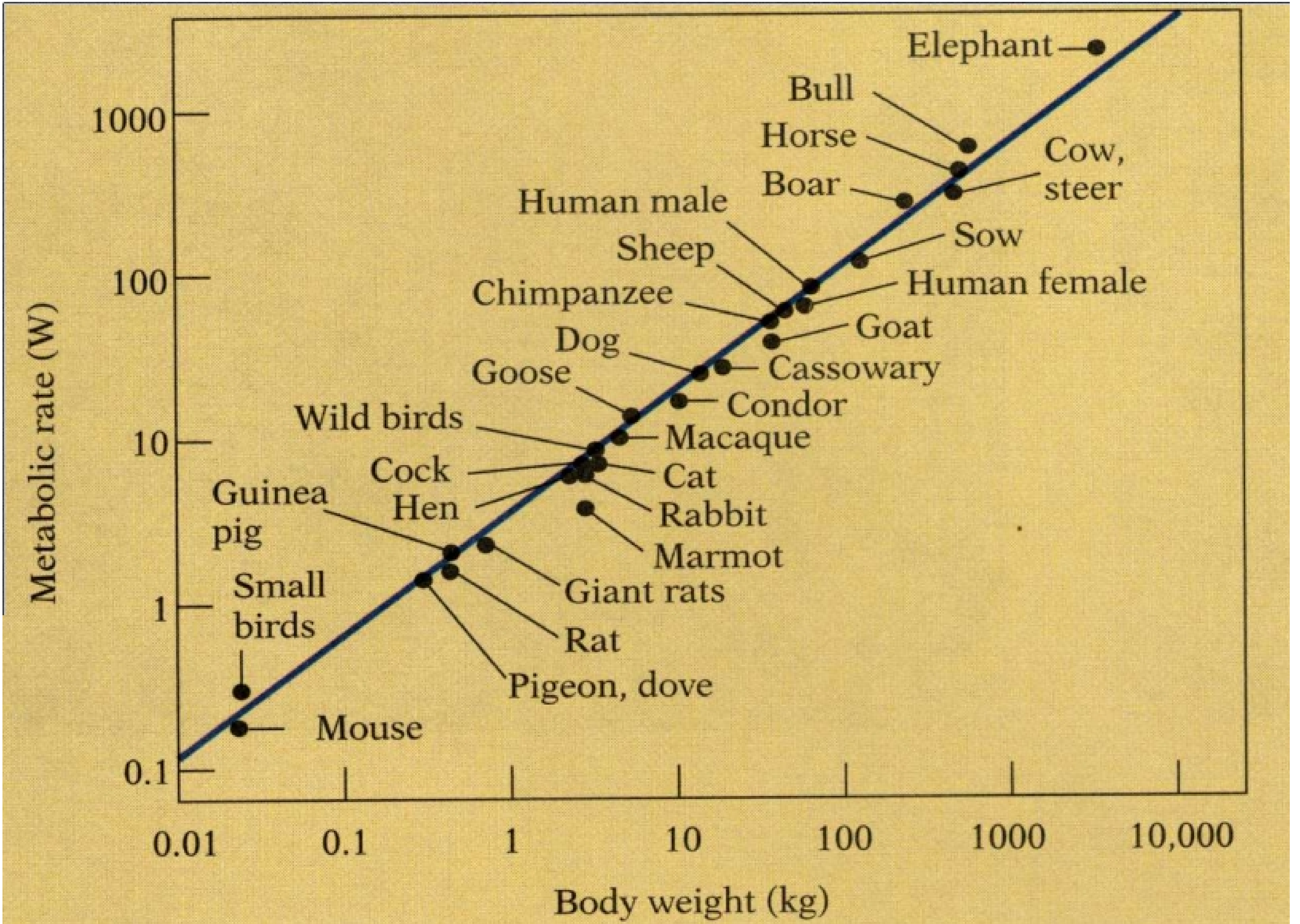
Hemmingson 1960

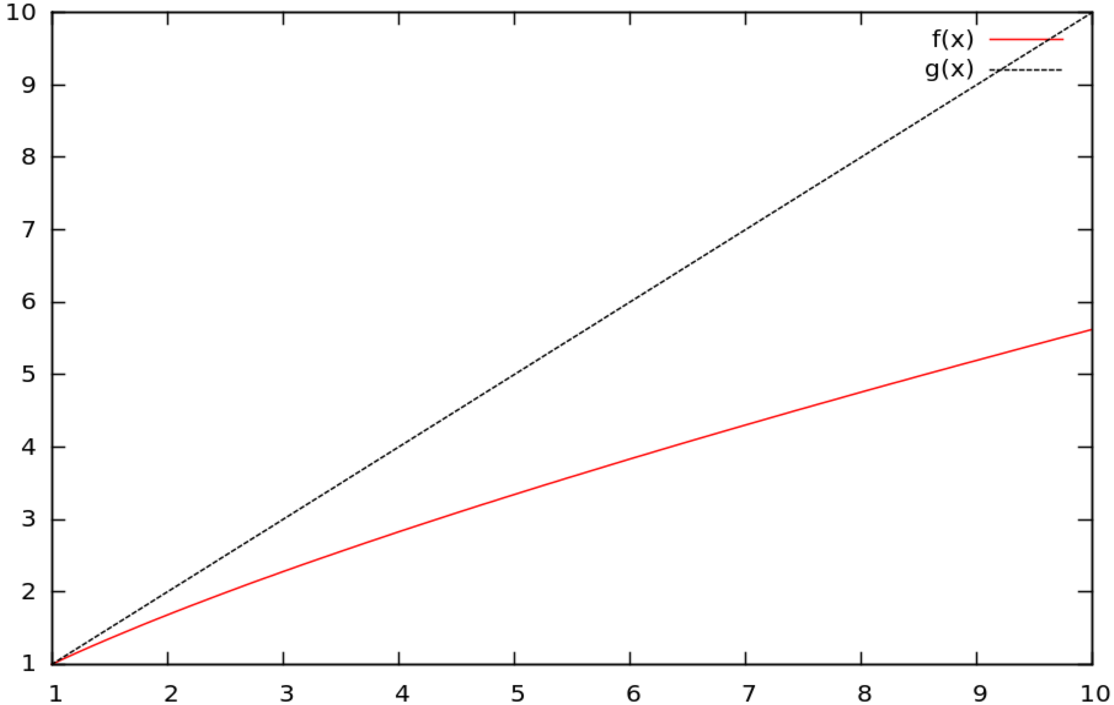


Muro: Limite imposto pelas Leis da Física;

Meninos: espécies biológicas;

Monstros: Seleção Natural;

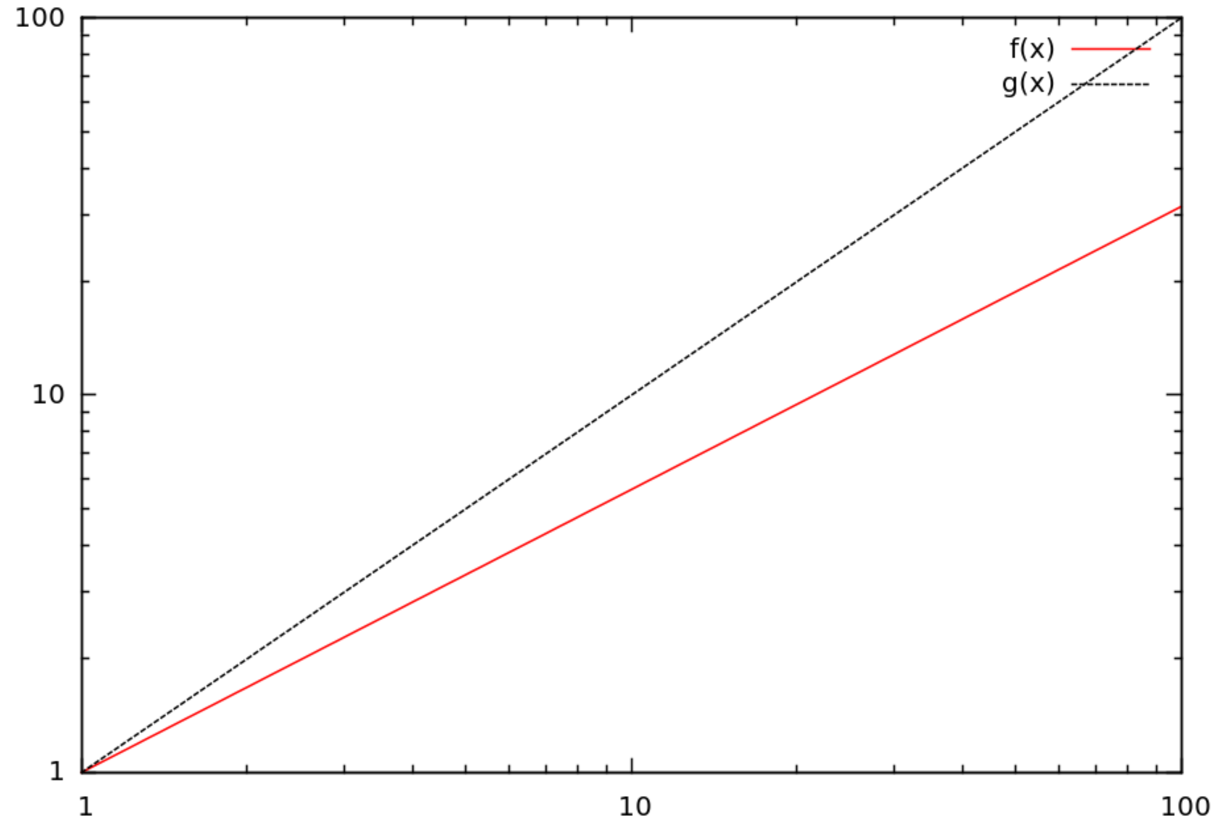

Elephant
Bull
Horse
Cow,
steer
Boar
Human male
Sow
Sheep
Human female
Chimpanzee
Goat
Dog
Cassowary
Goose
Condor
Wild birds
Macaque
Cock
Cat
Hen
Rabbit
Guinea
pig
Marmot
Giant rats
Small
birds
Rat
Pigeon, dove
Mouse
Metabolic rate (W)
1000
100
10
1
0.1
0.01
0.1
1
10
100
1000
10,000
Body weight (kg)



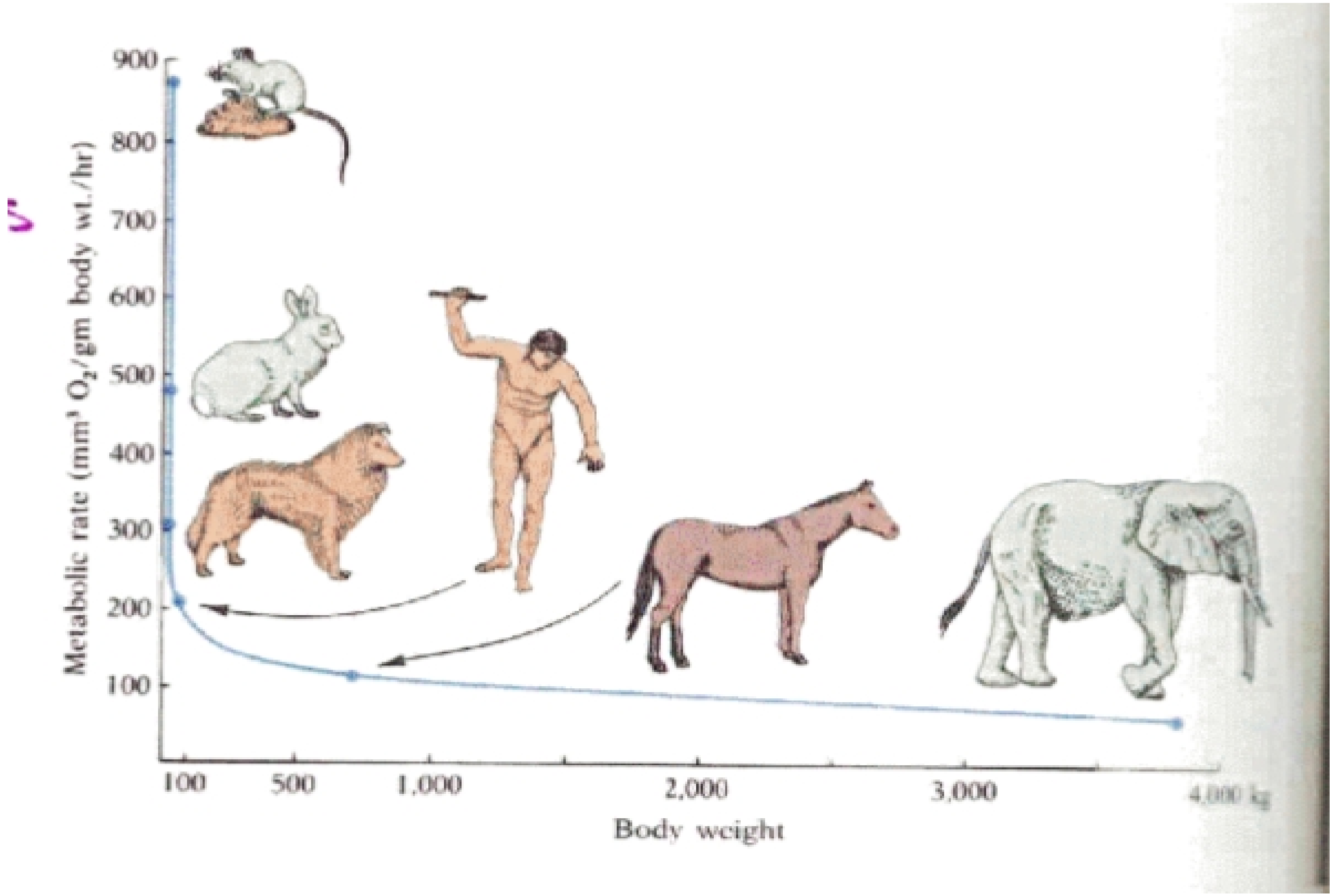
$$g(x) = x$$

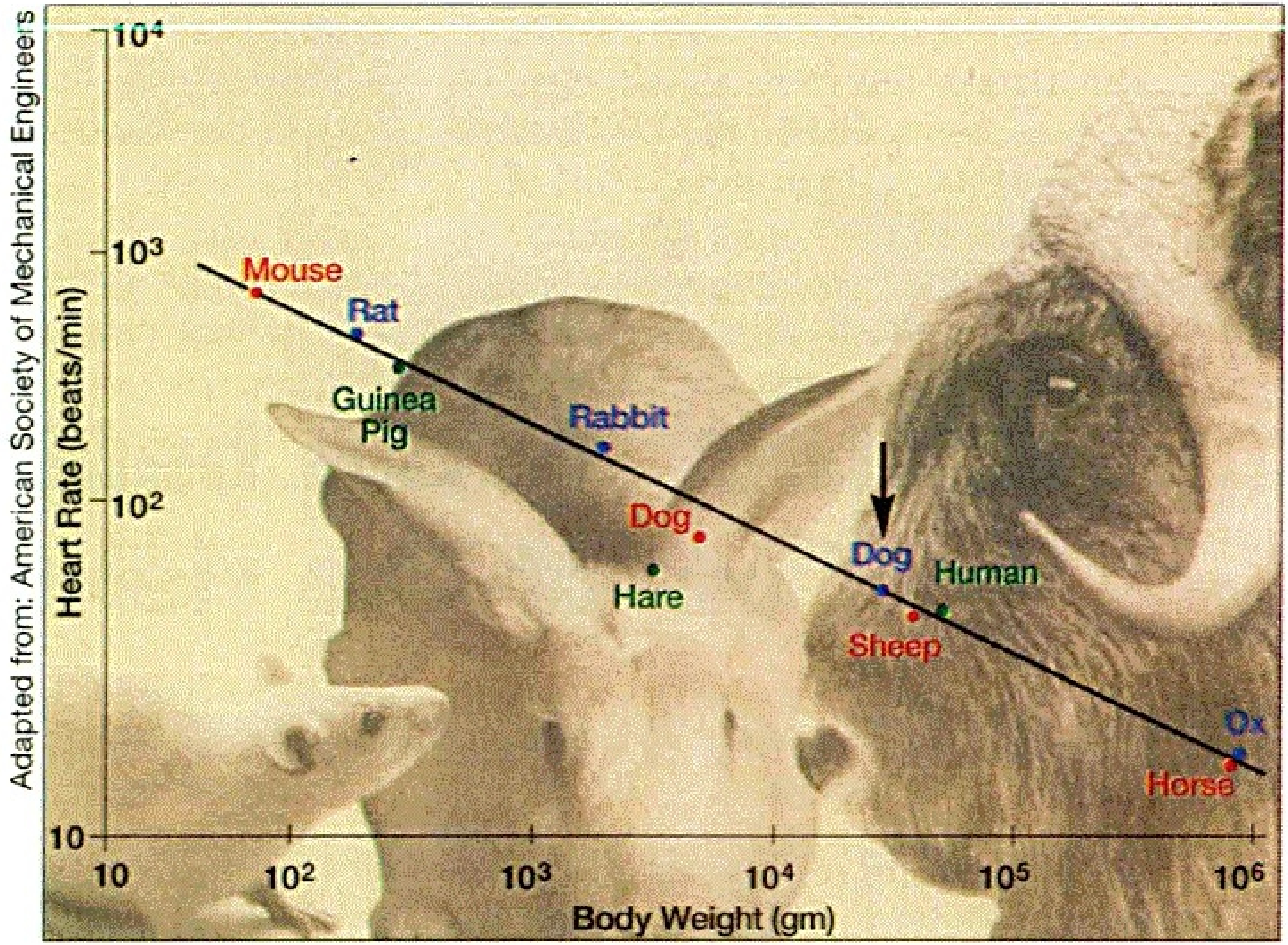
$$f(x) = x^{3/4}$$

Economia Energética com a Escala!

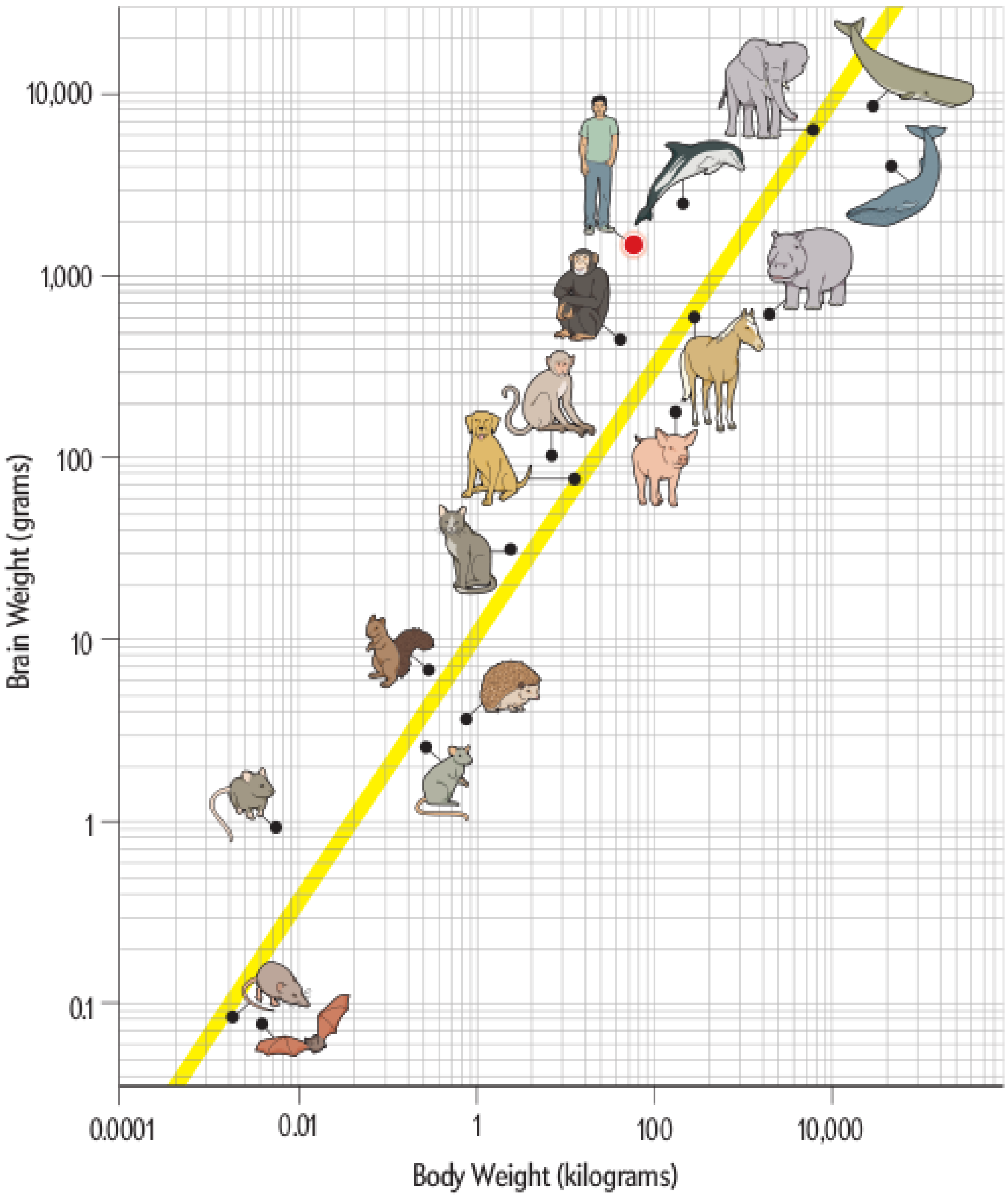


Economia Energética com a Escala!





Small mammals live fast and die young compared to big ones. Because heart rate tracks weight by a 1/4-power law, a dog (arrow) about 1/16 as heavy as a horse has a pulse about twice as fast as the horse's, not 16 times faster.



10,000
1,000
100
10
1
0.1
Brain Weight (grams)
0.0001
0.01
1
100
10,000
Body Weight (kilograms)

Energia total Metabolizada = Manutenção (das celulas existentes) + Crescimento (de novas células)

Energia total Metabolizada = Manutenção + Crescimento

(das celulas existentes) (de novas células)

$$B_0 \, m^{\beta} =$$

Energia total Metabolizada = Manutenção + Crescimento

(das celulas existentes) (de novas células)

$$B_0\ m^{\beta} = B_c\ m +$$

Energia total Metabolizada = Manutenção (das celulas existentes) + Crescimento (de novas células)

$$B_0\, m^{\beta} = B_c\, m + E_0\, dm/dt$$

Energia total Metabolizada = Manutenção + Crescimento

(das celulas existentes) (de novas células)

$$B_0 \, m^{\beta} = B_c \, m + E_0 \, dm/dt$$

$$\beta = 3/4 < 1$$

(escala sub-linear)

Energia total Metabolizada = Manutenção (das celulas existentes) + Crescimento (de novas células)

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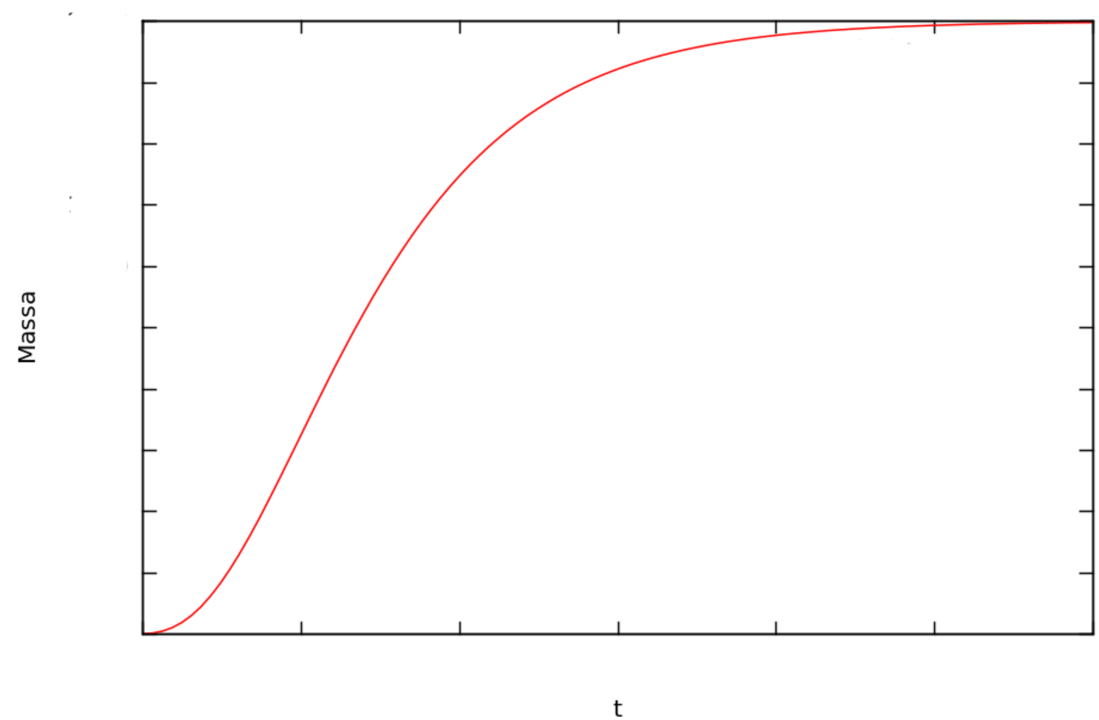
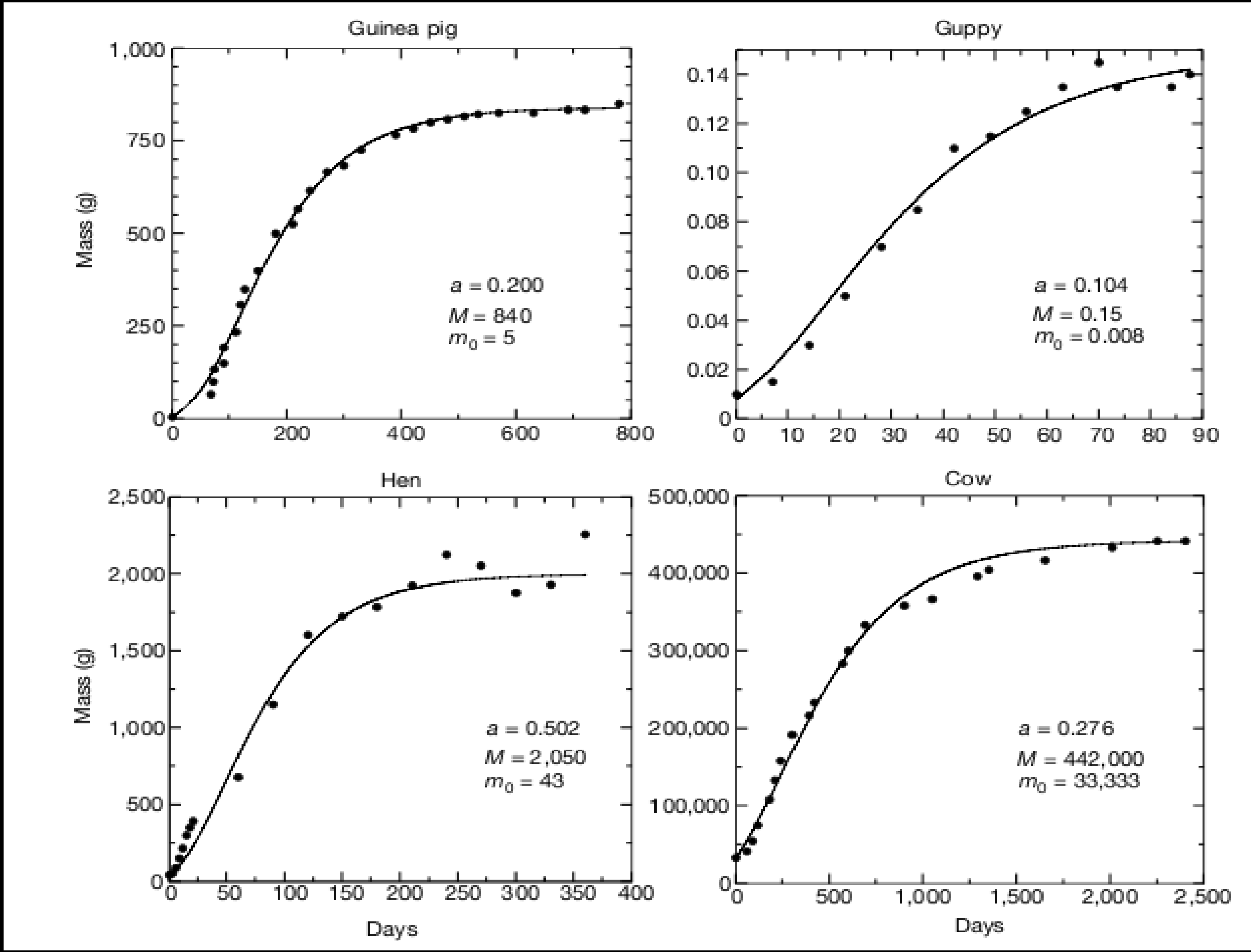
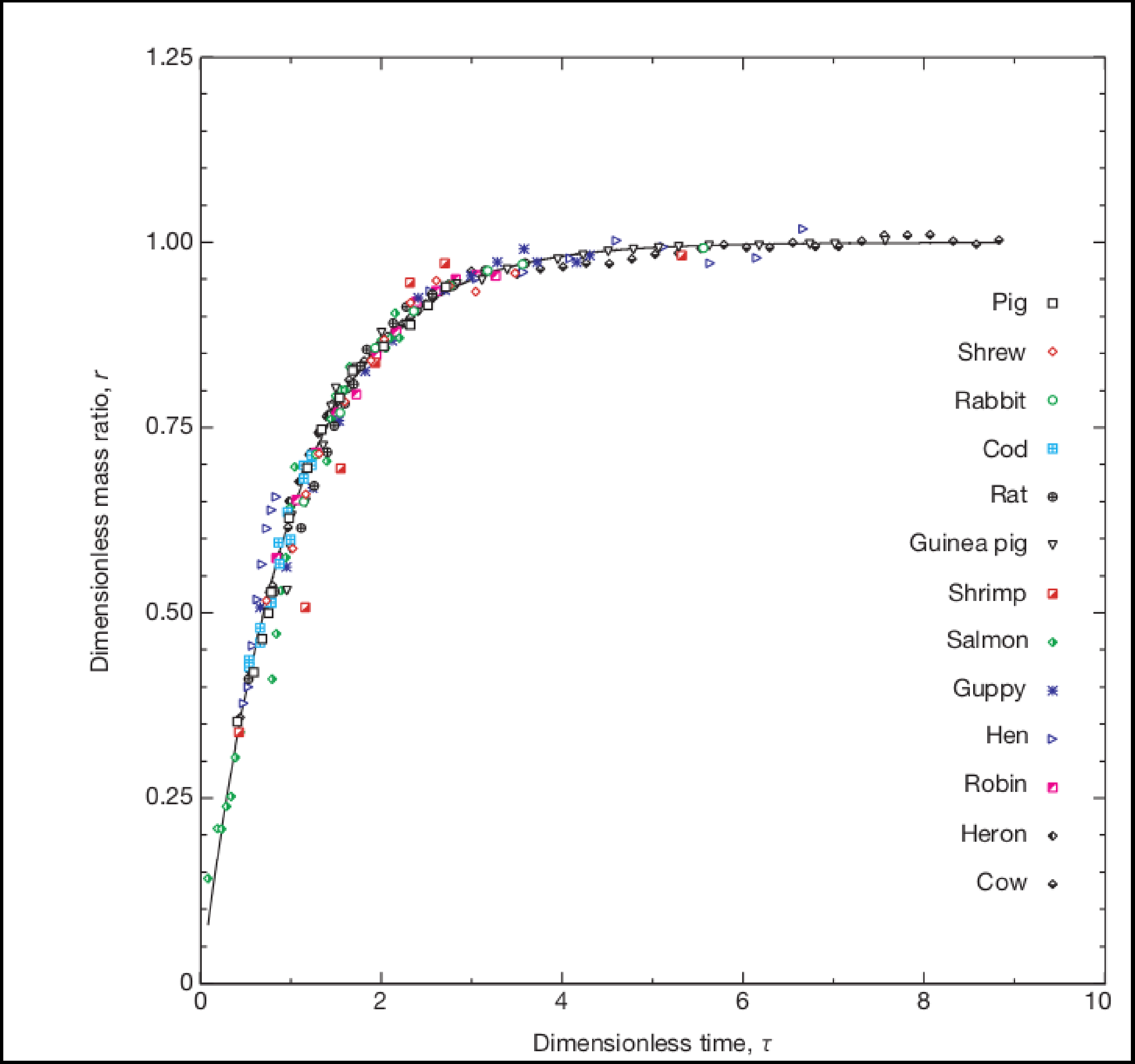
(escala sub-linear)





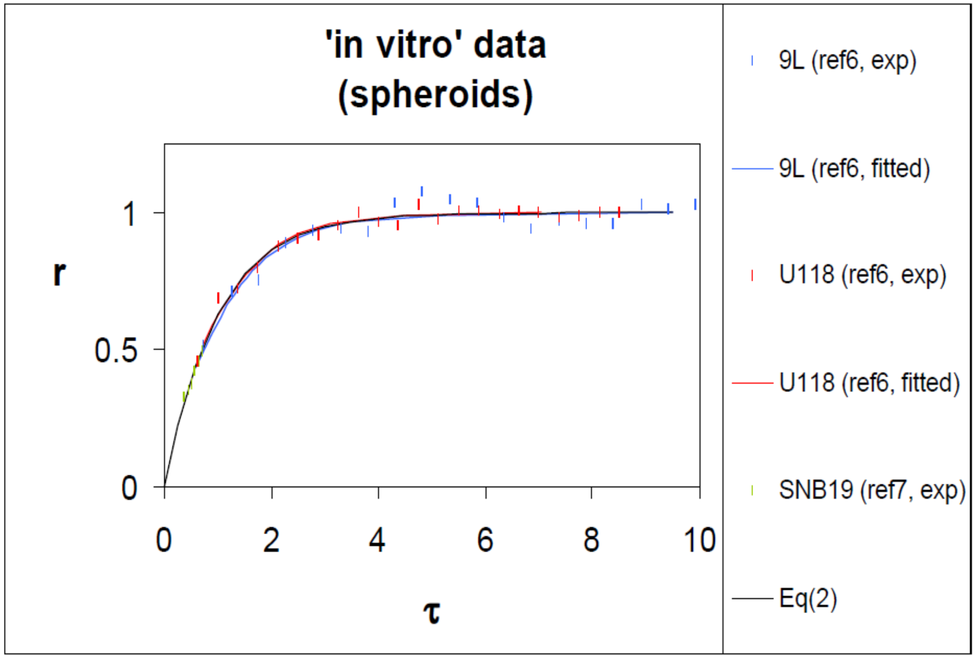
G. West et al , Nature Vol 413 (2001)

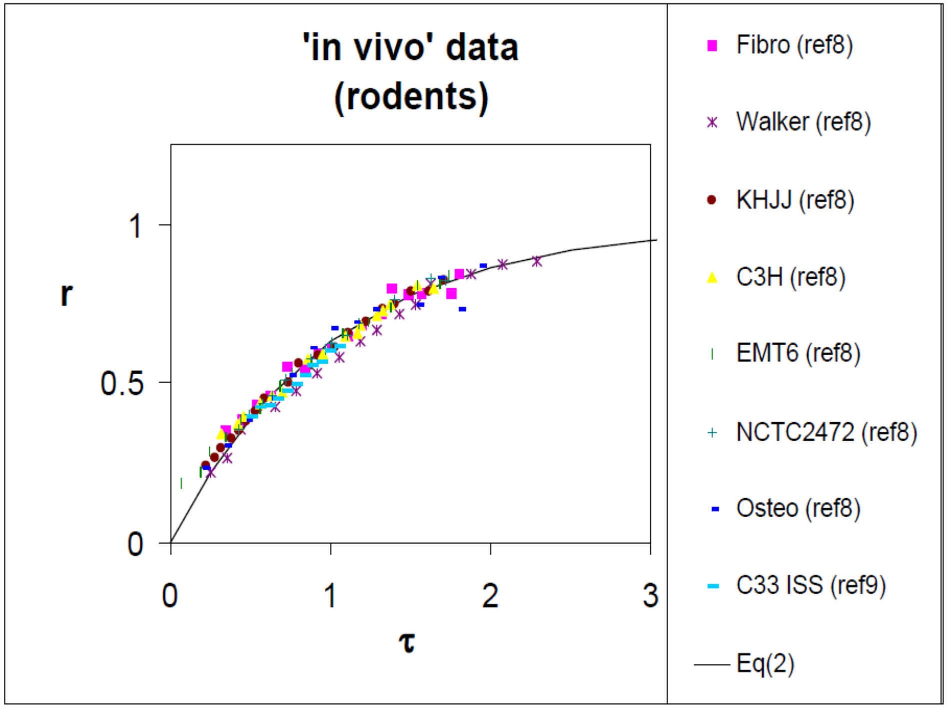


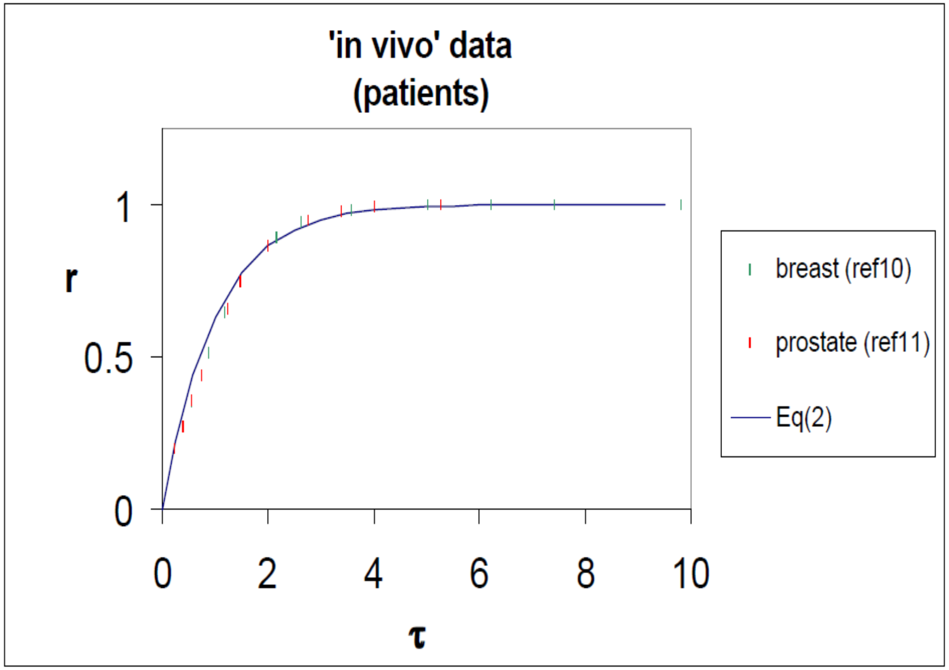
G. West et al , Nature Vol 413 (2001)

Curva de Crescimento de Tumor

Guiot et al , J. Theor. Biology 225, 147 (2003)







Leis de Escala na População Humana?

Francisco Morato
BR 381
Igaratá
SP-300
SP-065
Cabreúva
Santa Isabel
BR 116
Franco da Rocha
Mairiporã
SP-312
Cajamar
Arujá
BR 374
Santana de Parnaíba
Guararema
SP-348
Guarulhos
SP-070
SP-066
Barueri
Carapicuíba
SP-060
São Roque
Itapevi
Osasco
São Paulo
Suzano
Mogi das Cruzes
Cotia
Santo André
SP-031
Embu
São Bernardo do Campo
SP-098
Itapecerica da Serra
Ribeirão Pires
Salesópolis
SP-055
São Lourenço da Serra
Embu Guaçu
Bertioga
Cubatão
Juquitiba
SP-055
Santos
Praia Grande
Guarujá

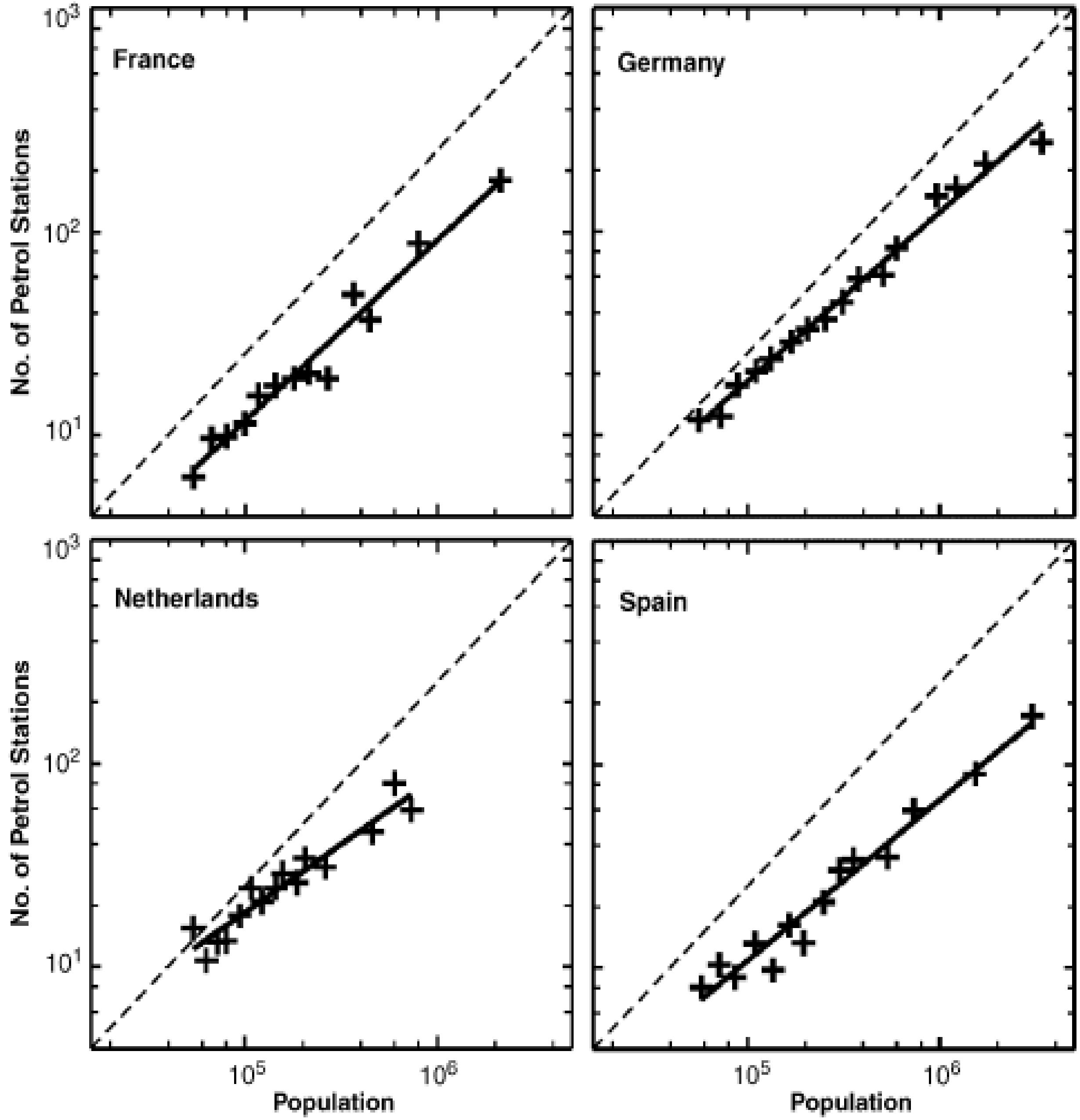
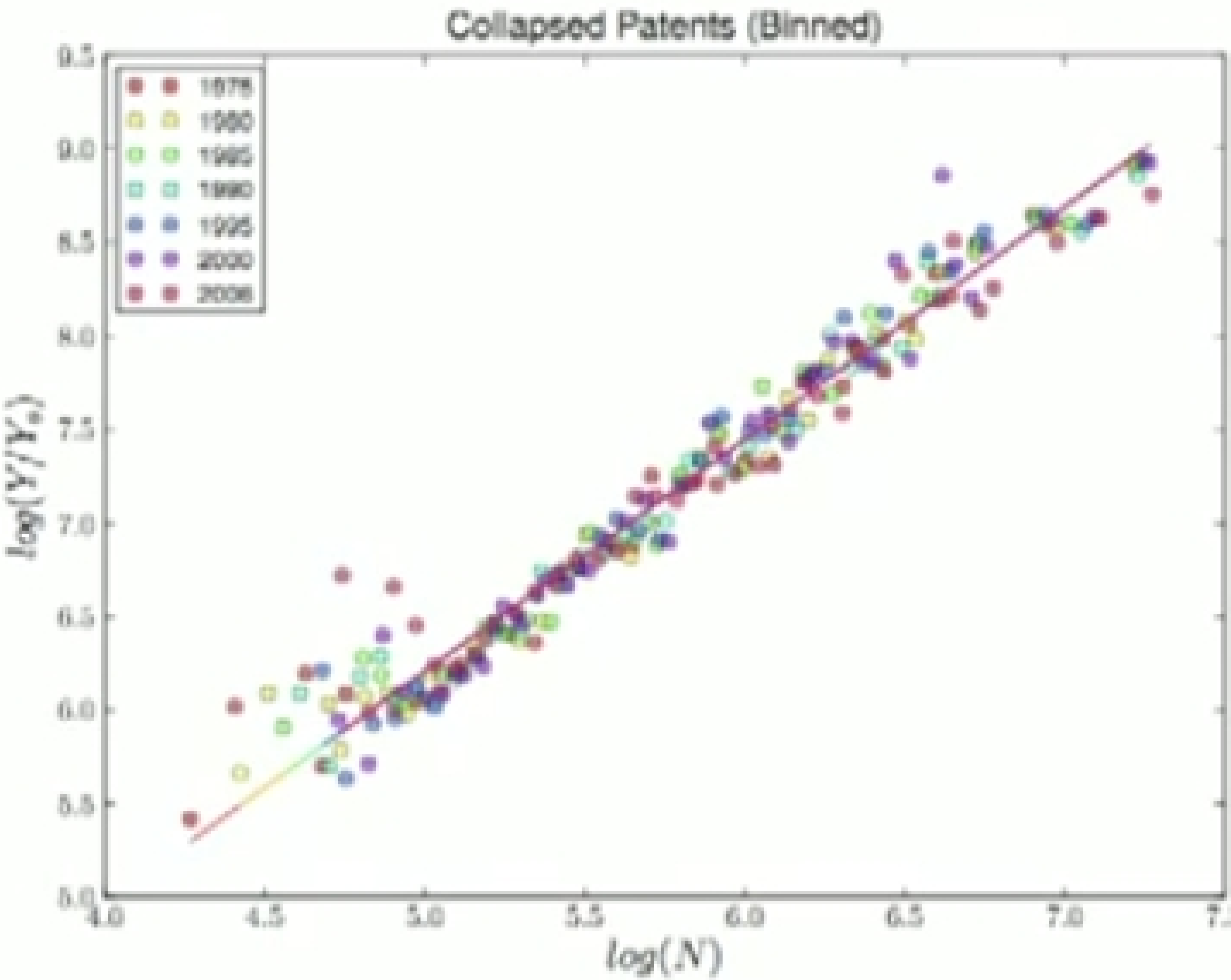
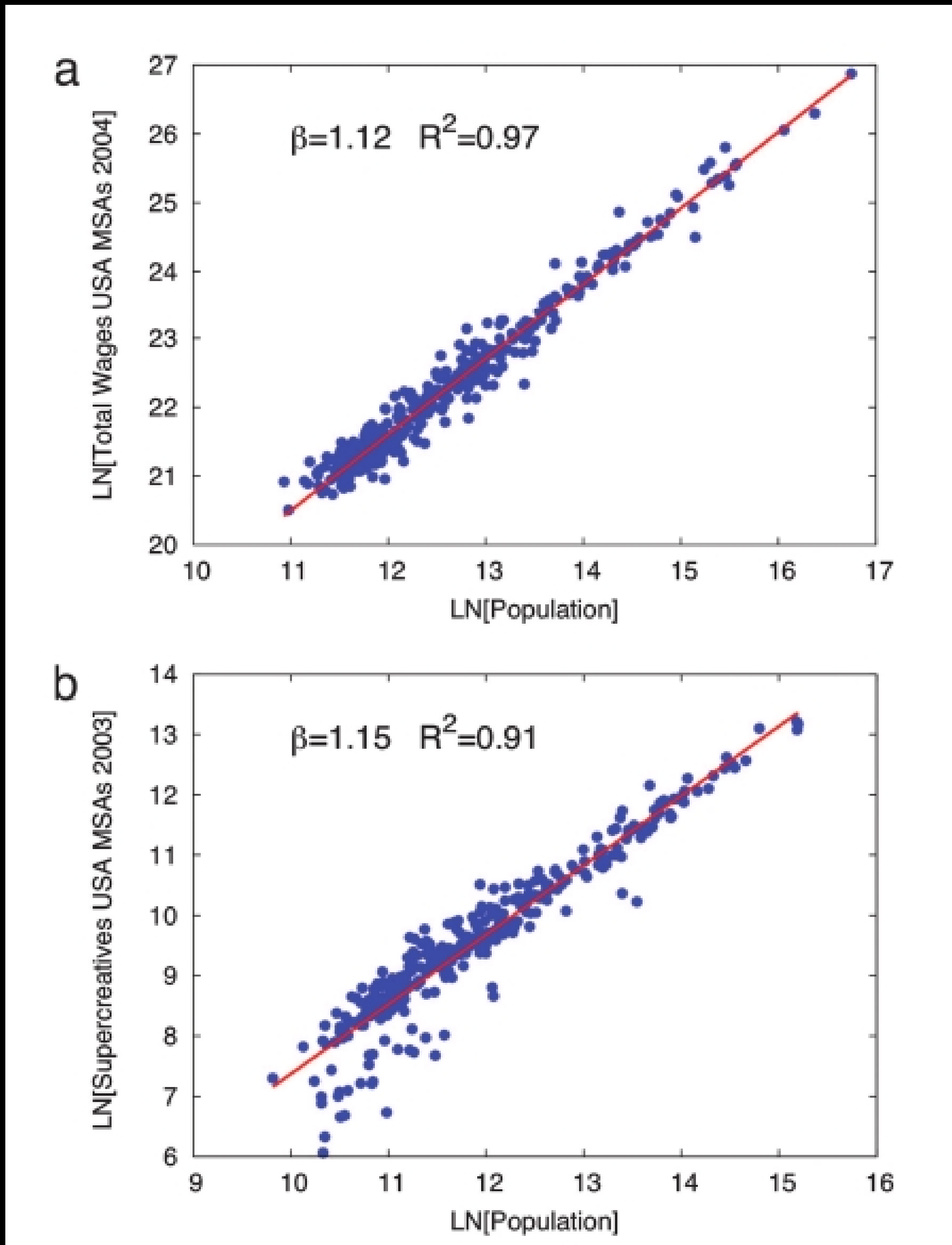


Fig. 4. Double-logarithmic representation of the number of petrol stations as a function of the population size of cities of France, Germany, Netherlands and Spain, after a logarithmic binning method has been applied. The solid lines correspond to the respective linear regression and the dashed lines indicate the slope 1.

Innovation measured by Patents





Example of scaling relationships

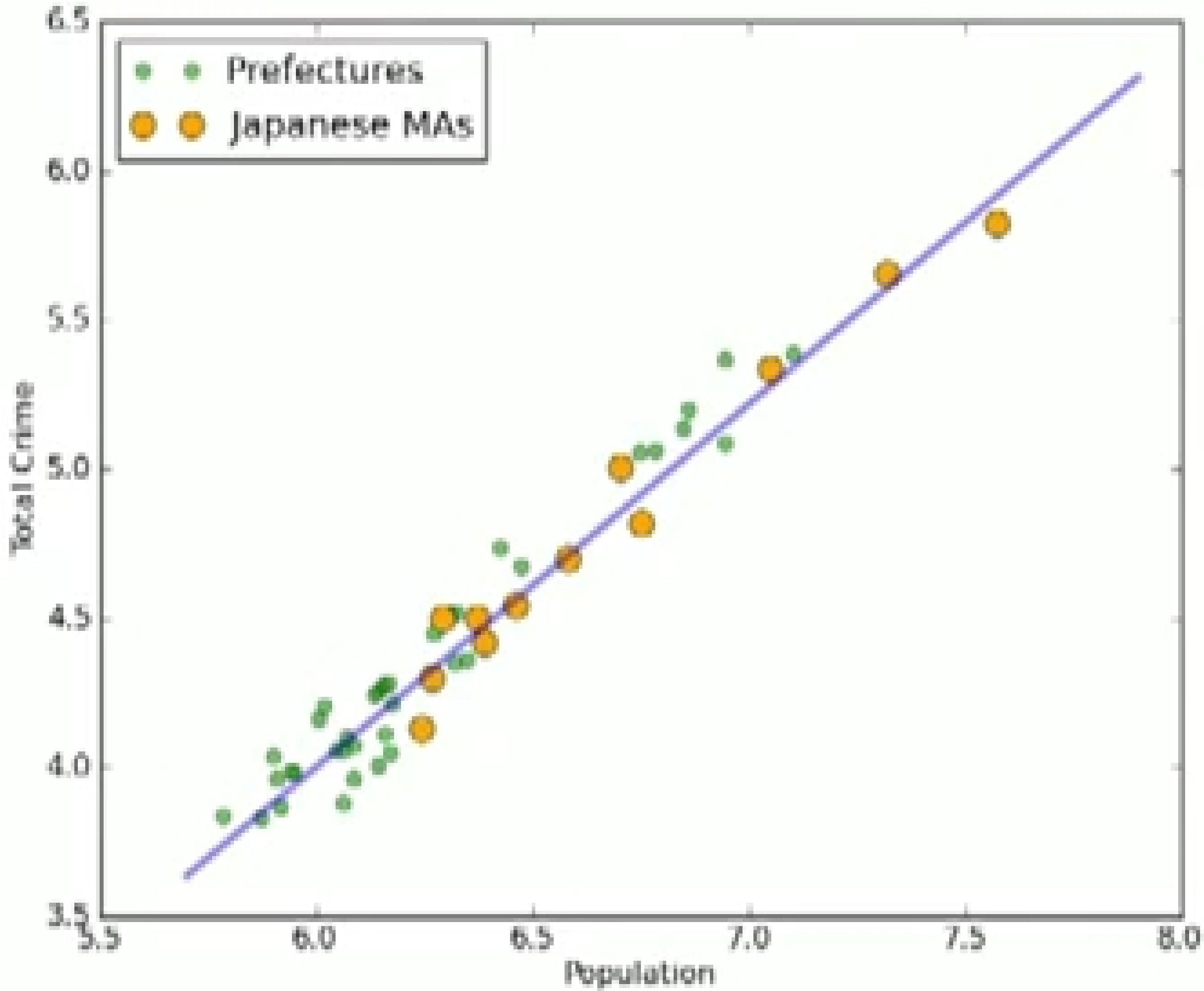
a) Total **WAGES** per MSA in 2004 for the USA vs. metropolitan population.

b) **SUPERCREATIVE** employment per MSA in 2003, for the USA vs. metropolitan population.

SUPER-LINEAR SCALING

L. Bettencourt et al , Pnas Vol. 104 (2007)

TOTAL CRIME (JAPAN)



Slope = 1.21 [1.08, 1.35]

$\beta > 1$

Interação

Social

Table 1. Scaling exponents for urban indicators vs. city size

Y	β	95% CI	Adj-R^2	Observations	Country–year
New patents	1.27	[1.25,1.29]	0.72	331	U.S. 2001
Inventors	1.25	[1.22,1.27]	0.76	331	U.S. 2001
Private R&D employment	1.34	[1.29,1.39]	0.92	266	U.S. 2002
"Supercreative" employment	1.15	[1.11,1.18]	0.89	287	U.S. 2003
R&D establishments	1.19	[1.14,1.22]	0.77	287	U.S. 1997
R&D employment	1.26	[1.18,1.43]	0.93	295	China 2002
Total wages	1.12	[1.09,1.13]	0.96	361	U.S. 2002
Total bank deposits	1.08	[1.03,1.11]	0.91	267	U.S. 1996
GDP	1.15	[1.06,1.23]	0.96	295	China 2002
GDP	1.26	[1.09,1.46]	0.64	196	EU 1999–2003
GDP	1.13	[1.03,1.23]	0.94	37	Germany 2003
Total electrical consumption	1.07	[1.03,1.11]	0.88	392	Germany 2002
New AIDS cases	1.23	[1.18,1.29]	0.76	93	U.S. 2002–2003
Serious crimes	1.16	[1.11, 1.18]	0.89	287	U.S. 2003

L. Bettencourt et al , Pnas Vol. 104 (2007)

β> 1

Interação

Social

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Serious crimes	1.16	[1.11, 1.18]	0.89	287	U.S. 2003

β = 1

Necessidades individuais

Total housing	1.00	[0.99,1.01]	0.99	316	U.S. 1990
Total employment	1.01	[0.99,1.02]	0.98	331	U.S. 2001
Household electrical consumption	1.00	[0.94,1.06]	0.88	377	Germany 2002
Household electrical consumption	1.05	[0.89,1.22]	0.91	295	China 2002
Household water consumption	1.01	[0.89,1.11]	0.96	295	China 2002

L. Bettencourt et al , Pnas Vol. 104 (2007)

β> 1

Interação Social

β = 1

Necessidades individuais

β < 1

Infro-estrutura

Table 1. Scaling exponents for urban indicators vs. city size

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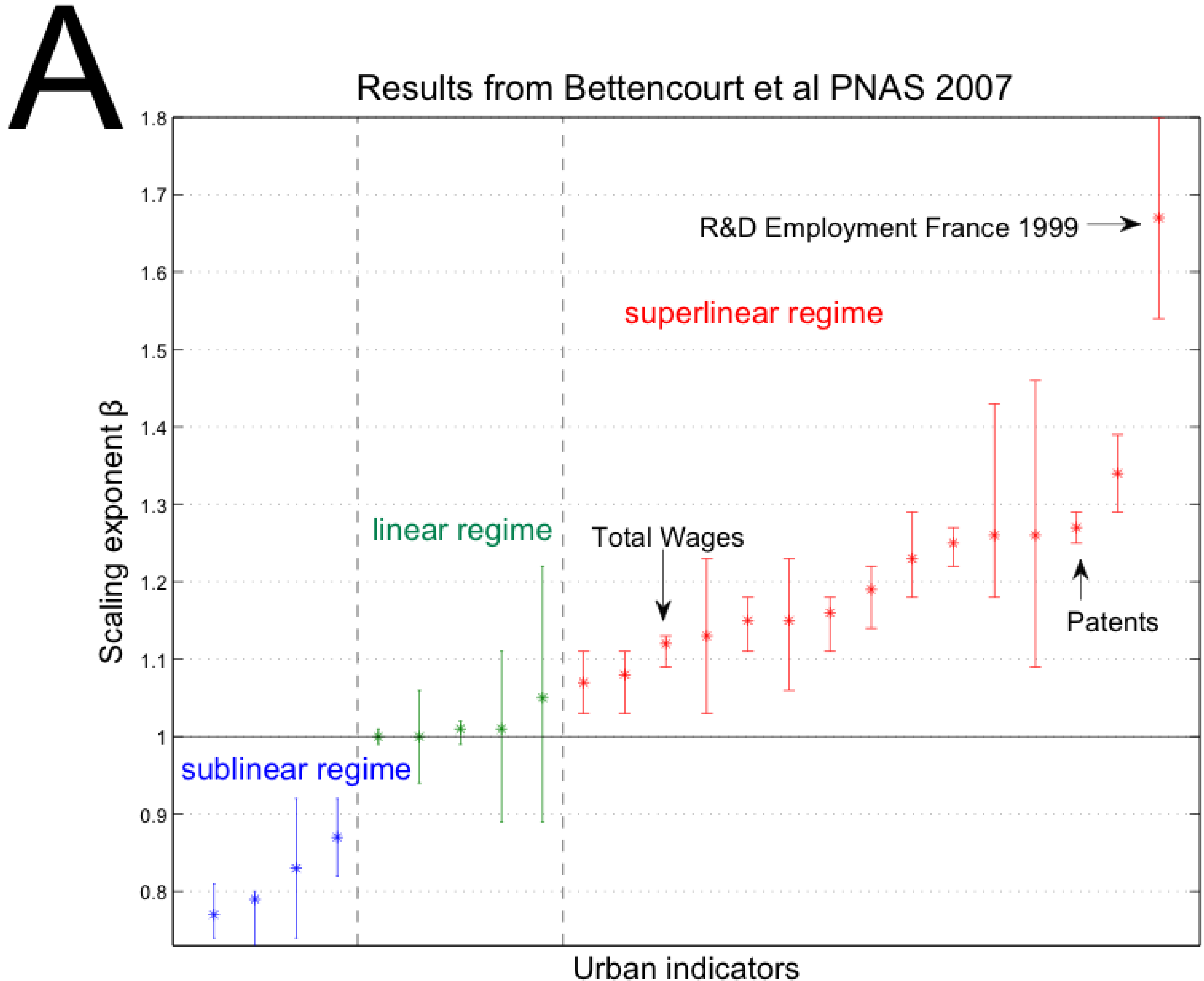
Household water consumption	1.01	[0.89,1.11]	0.96	295	China 2002
Gasoline stations	0.77	[0.74,0.81]	0.93	318	U.S. 2001
Gasoline sales	0.79	[0.73,0.80]	0.94	318	U.S. 2001
Length of electrical cables	0.87	[0.82,0.92]	0.75	380	Germany 2002
Road surface	0.83	[0.74,0.92]	0.87	29	Germany 2002

Data sources are shown in *SI Text*. CI, confidence interval; Adj-R^2, adjusted R^2; GDP, gross domestic product.

L. Bettencourt et al , Pnas Vol. 104 (2007)



A
Results from Bettencourt et al PNAS 2007
1.8
1.7
1.6
1.5
1.4
1.3
1.2
1.1
1
0.9
0.8
Scaling exponent β
Urban indicators
R&D Employment France 1999
superlinear regime
linear regime
Total Wages
Patents
sublinear regime

Duplicando o tamanho da cidade

Aumenta-se sistematicamente

(aproximadamente 15%):

-

Duplicando o tamanho da cidade

Aumenta-se sistematicamente

(aproximadamente 15%):

- Salários;
- Riquezas;
- Número de Patentes;
- Número de Pessoas Criativas;

Duplicando o tamanho da cidade

Aumenta-se sistematicamente

(aproximadamente 15%):

- Salários;
- Riquezas;
- Número de Patentes;
- Número de Pessoas Criativas;

- Número de Policiais;
- Número de crimes;
- Número de casos de AIDS e outros;

Recusos Disponíveis = Manutenção + Crescimento

(dos indivíduos existentes) (de novos individuos)

$$\text{Recusos Disponíveis} = \text{Manutenção} + \text{Crescimento}$$

(dos indivíduos existentes) (de novos individuos)

$$R_0 N^{\beta} =$$

Recusos Disponíveis = Manutenção + Crescimento

(dos indivíduos existentes) (de novos individuos)

$$R_0 N^{\beta} = R_c N +$$

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Recusos Disponíveis = Manutenção + Crescimento

(dos indivíduos existentes) (de novos individuos)

$$R_0 N^{\beta} = R_c N + E_c \, dN/dt$$

$$\beta > 1$$

(escala super-linear)

Recusos Disponíveis = Manutenção (dos indivíduos existentes) + Crescimento (de novos individuos)

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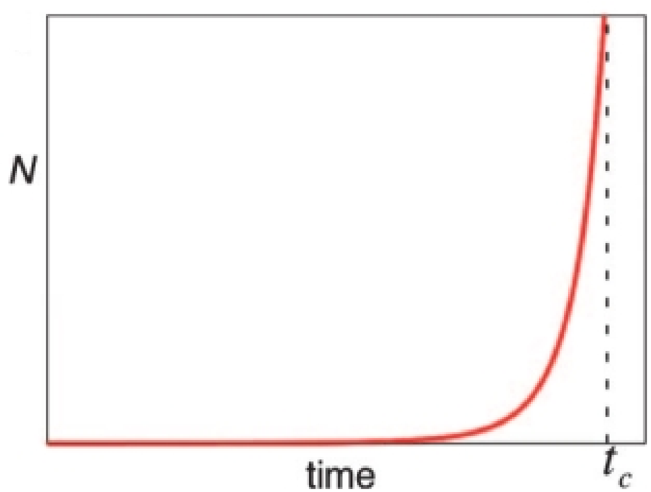
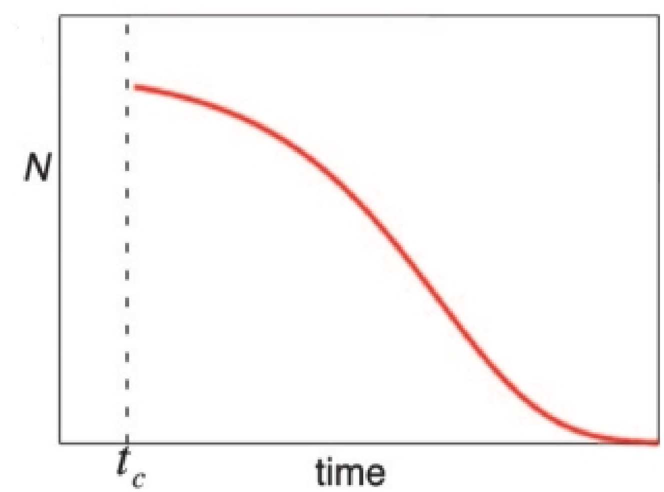
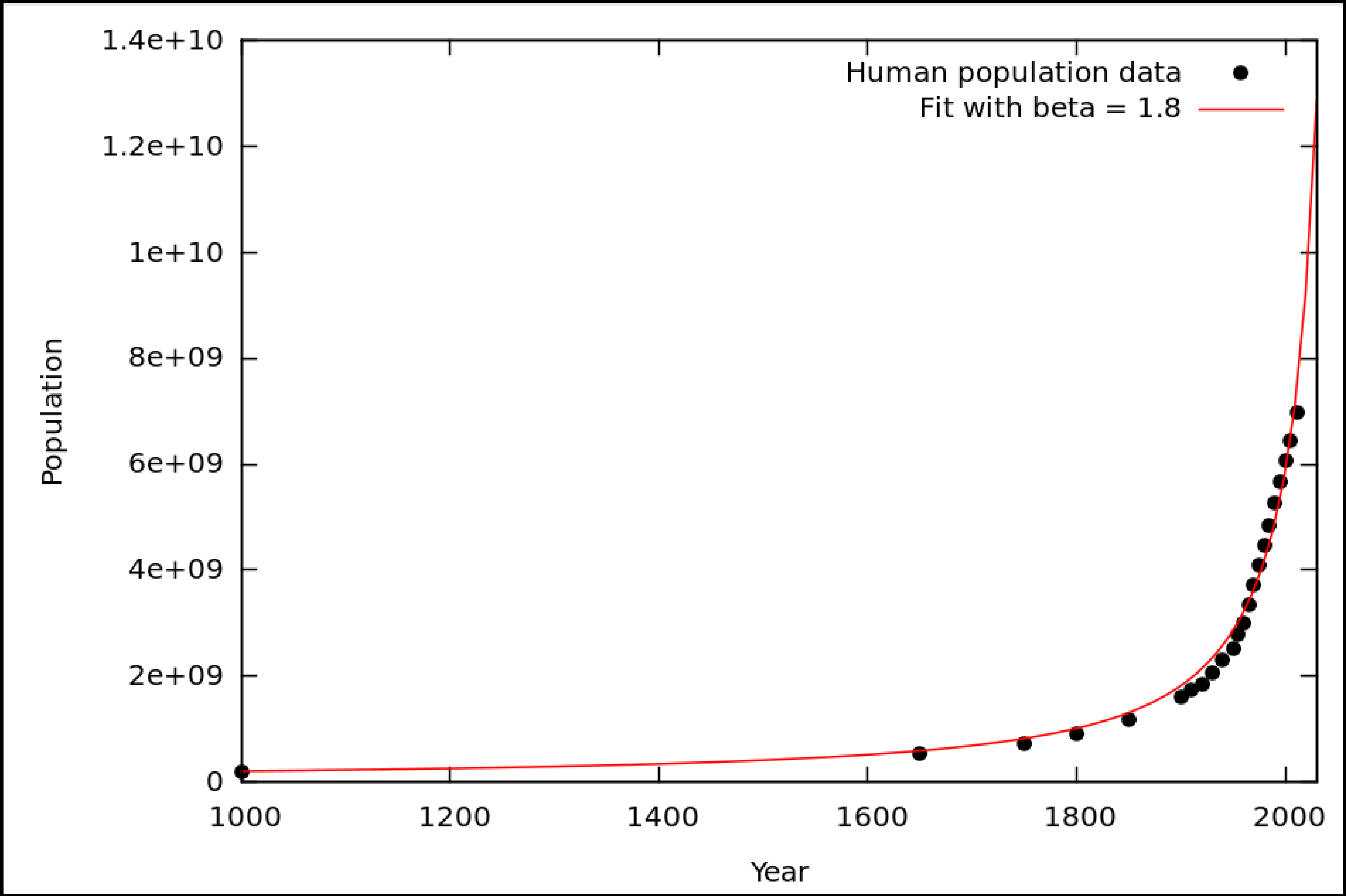
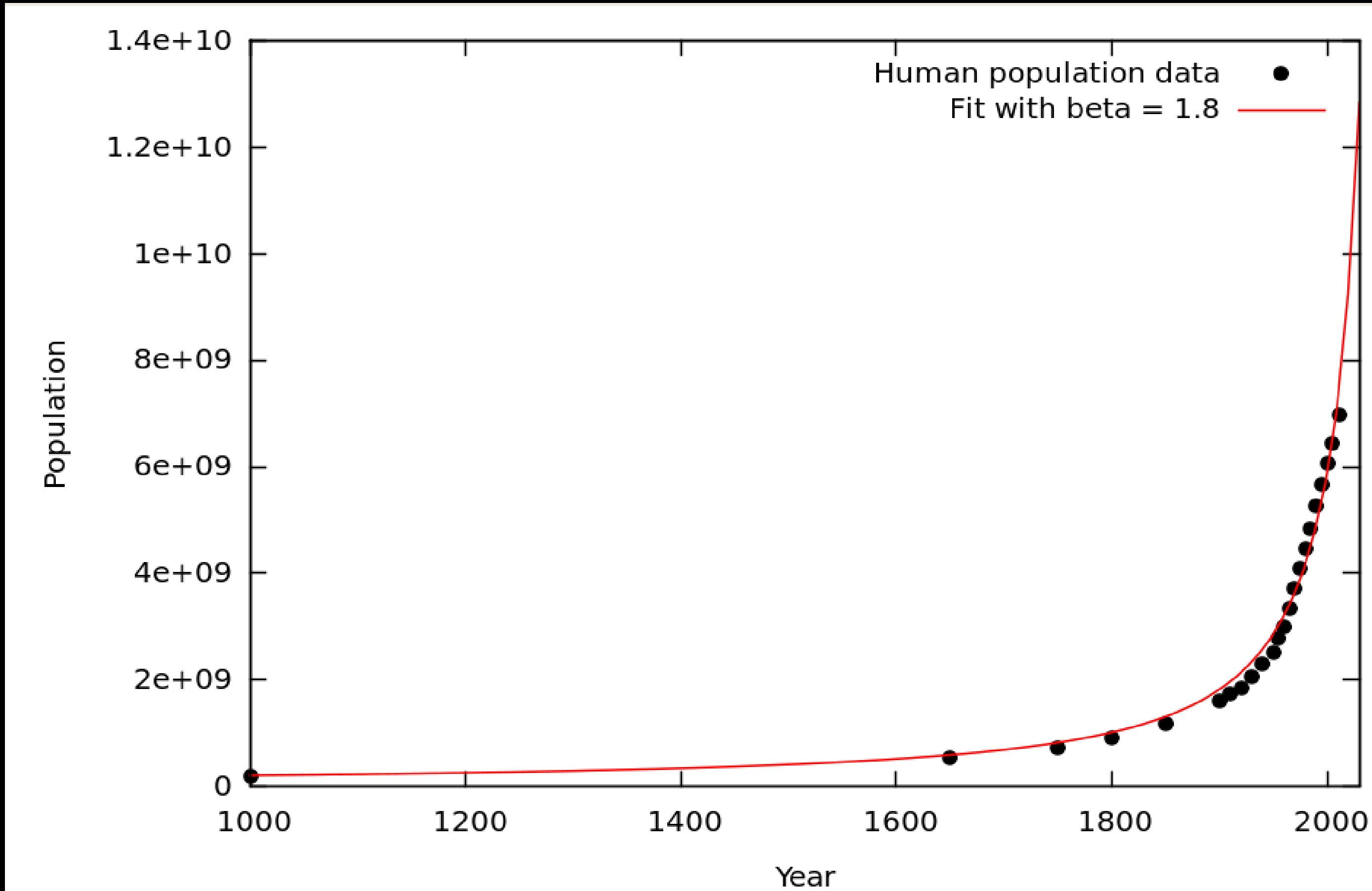
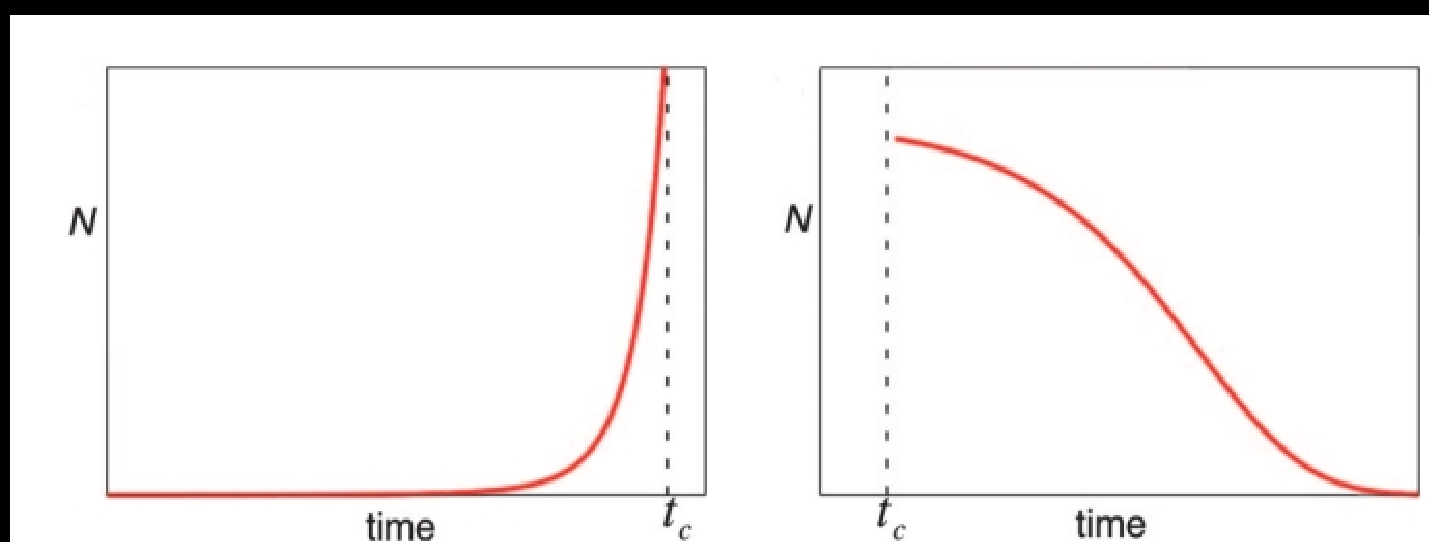






Human population data
Fit with beta = 1.8
Population
Year
1.4e+10
1.2e+10
1e+10
8e+09
6e+09
4e+09
2e+09
0
1000
1200
1400
1600
1800
2000


Human population data
Fit with beta = 1.8
Population
Year
1.4e+10
1.2e+10
1e+10
8e+09
6e+09
4e+09
2e+09
0
1000
1200
1400
1600
1800
2000


N
time
t_c
N
t_c
time

Doomsday: Friday, 13 November, A.D. 2026

At this date human population will approach infinity if it grows as it has grown in the last two millenia.

Heinz von Foerster, Patricia M. Mora, Lawrence W. Amiot

Among the many different aspects which may be of interest in the study of biological populations (*1*) is the one in which attempts are made to estimate the past and the future of such a population in terms of the number of its elements, if the behavior of this population is observable over a reasonable period of time.

All such attempts make use of two fundamental facts concerning an individual element of a closed biological population—namely, (i) that each element comes into existence by a sexual or asexual process performed by another element of this population ("birth"), and (ii) that after a finite time each element will cease to be a distinguishable member of this population and has to be excluded from the population count ("death").

Under conditions which come close to being paradise—that is, no environmental hazards, unlimited food supply, and no detrimental interaction between elements—the fate of a biological population as a whole is completely determined at all times by reference to the two fundamental properties of an individual element: its fertility and its mortality. Assume, for simplicity, a fictitious population in which all elements behave identically (equivivant population, *2*) displaying a fertility of γ_0 offspring per element per unit time and having a mortality $\theta_0 = 1/t_m$, derived from the life span for an individual element of t_m units of time. Clearly, the elements in the population, is given by

$$\frac{dN}{dt} = \gamma_0 N - \theta_0 N = a_0 N \qquad (1)$$

where $a_0 = \gamma_0 - \theta_0$ may be called the productivity of the individual element. Depending upon whether $a_0 \gtreqless 0$, integration of Eq. 1 gives the well-known exponential growth or decay of such a population with a time constant of $1/a_0$.

In reality, alas, the situation is not that simple, inasmuch as the two parameters describing fertility and mortality may vary from element to element and, moreover, fertility may have different values, depending on the age of a particular element.

To derive these distribution functions from observations of the behavior of a population as a whole involves the use of statistical machinery of considerable sophistication (*3*, *4*).

However, so long as the elements live in our hypothetical paradise, it is in principle possible, by straightforward mathematical methods, to extract the desired distribution functions, and the fate of the population as a whole, with all its ups and downs, is again determined by properties exclusively attributable to individual elements. If one foregoes the opportunity to describe the behavior of a population in all its temporal details and is satisfied

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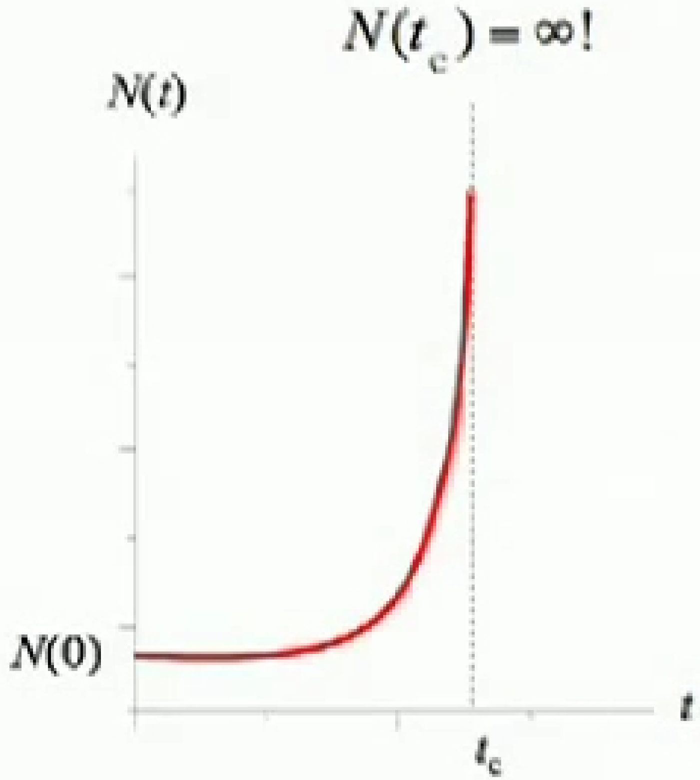
The authors are members of the staff of the department of electrical engineering, University of Illinois, Urbana.

b> 1 UNBOUNDED GROWTH UP TO FINITE TIME SINGULARITY



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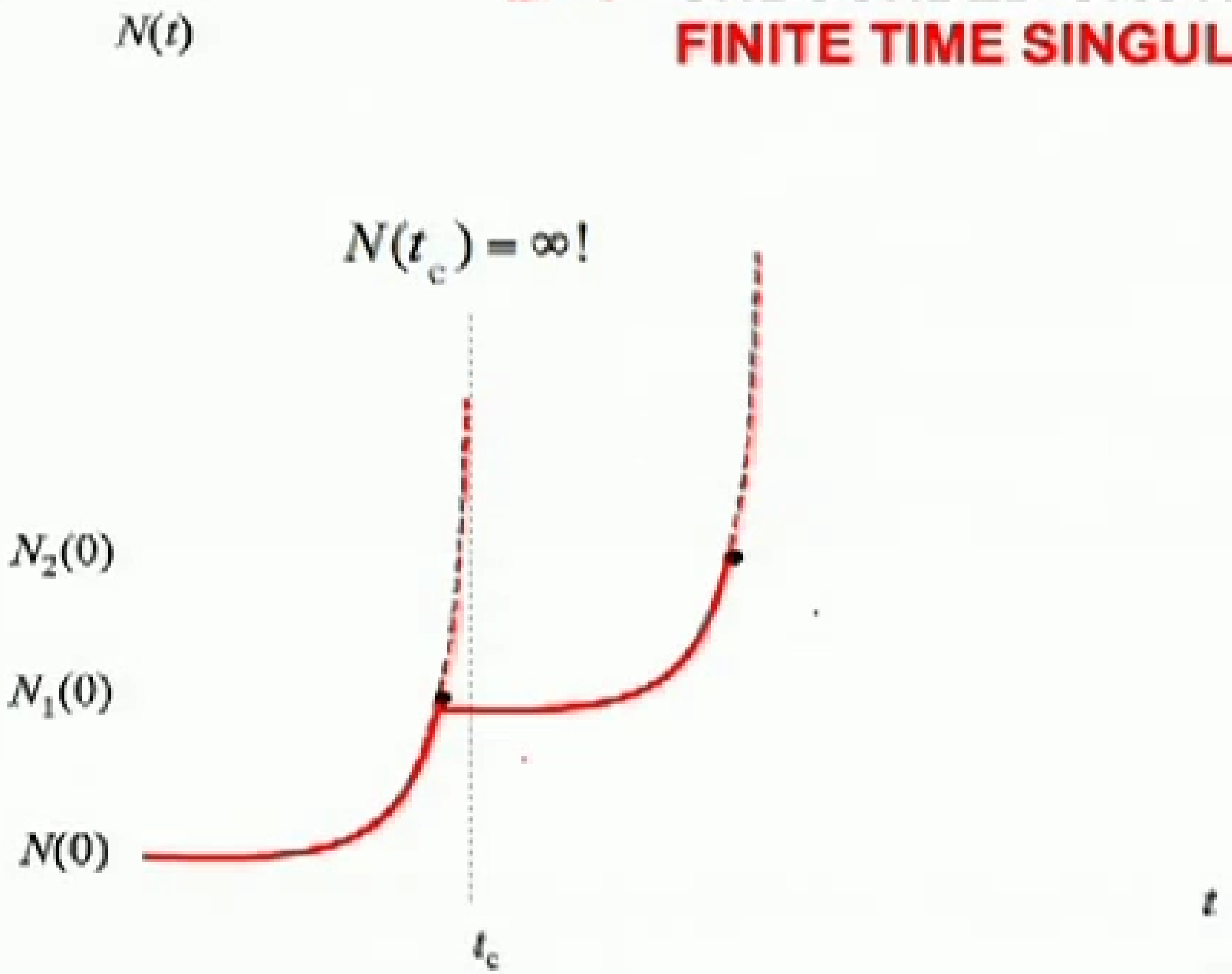

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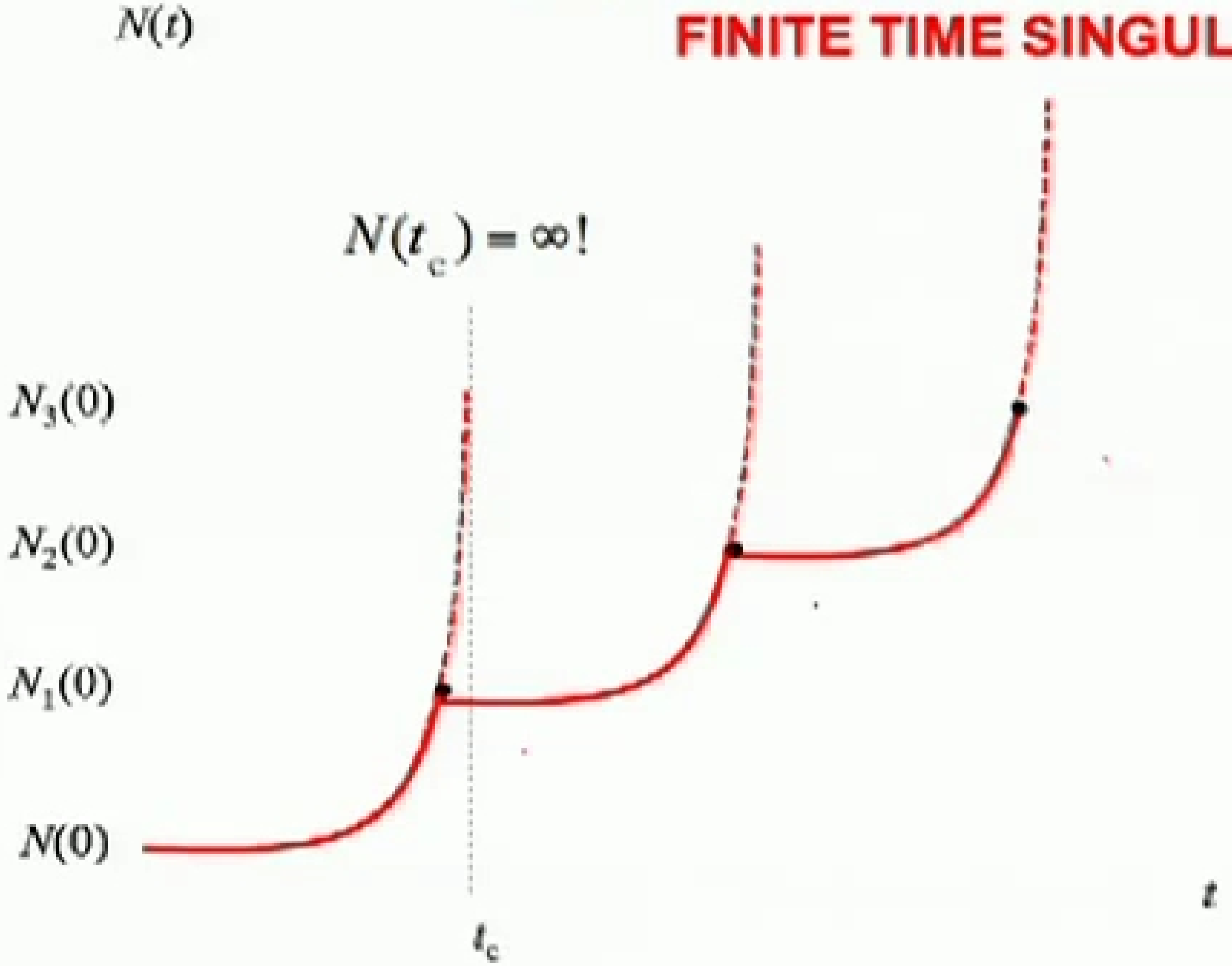
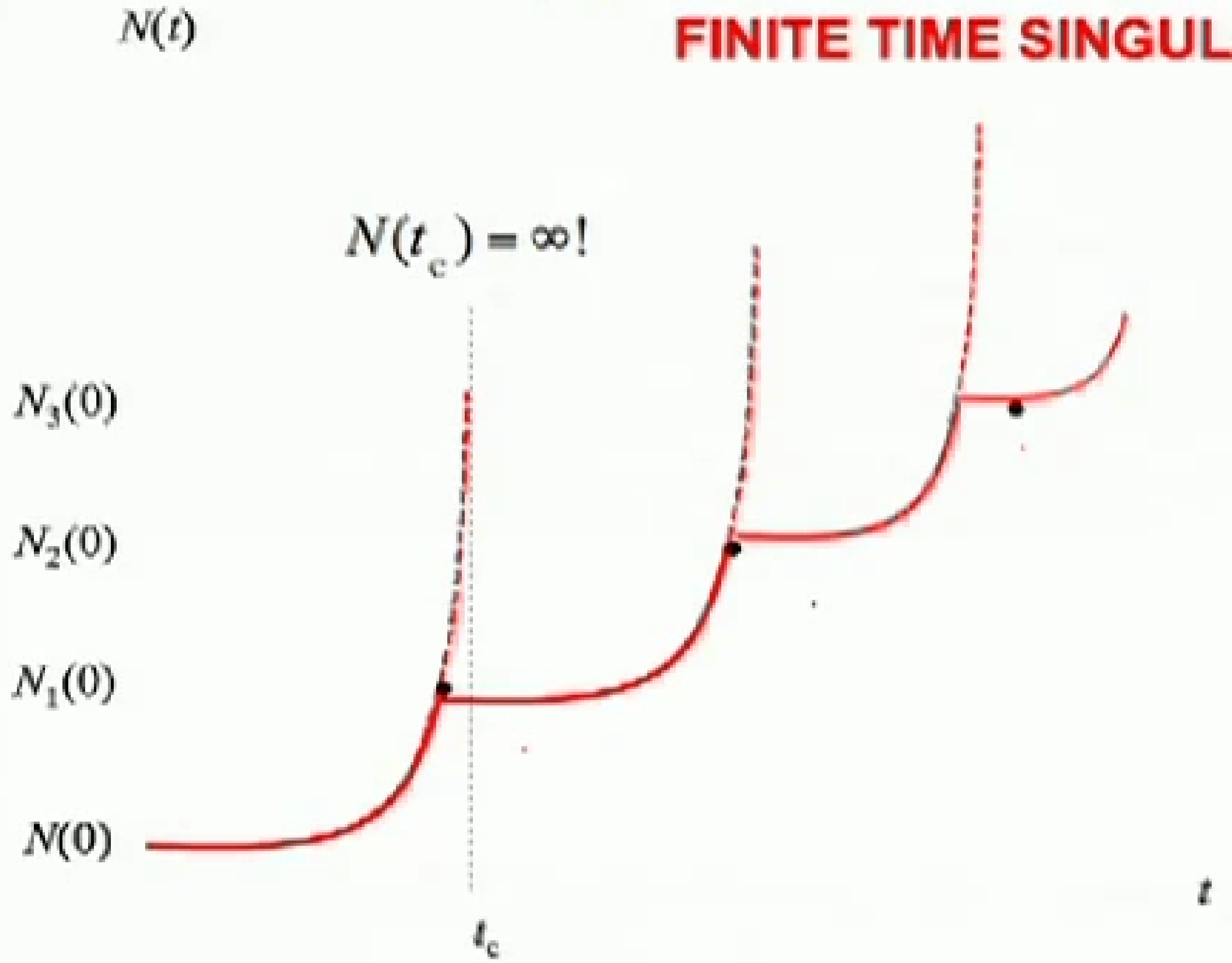


b> 1 UNBOUNDED GROWTH UP TO FINITE TIME SINGULARITY



Conclusões

$$c_1 N^{\beta} = c_2\, N + c_3\, dN/dt$$

- Interação Social: $\beta > 1$

  (escala super-linear)

- Biologia e Infro-estrutura de cidades: $\beta < 1$

  (escala sub-linear)

Conclusões

$$c_1 N^{\beta} = c_2 N + c_3 \, dN/dt$$