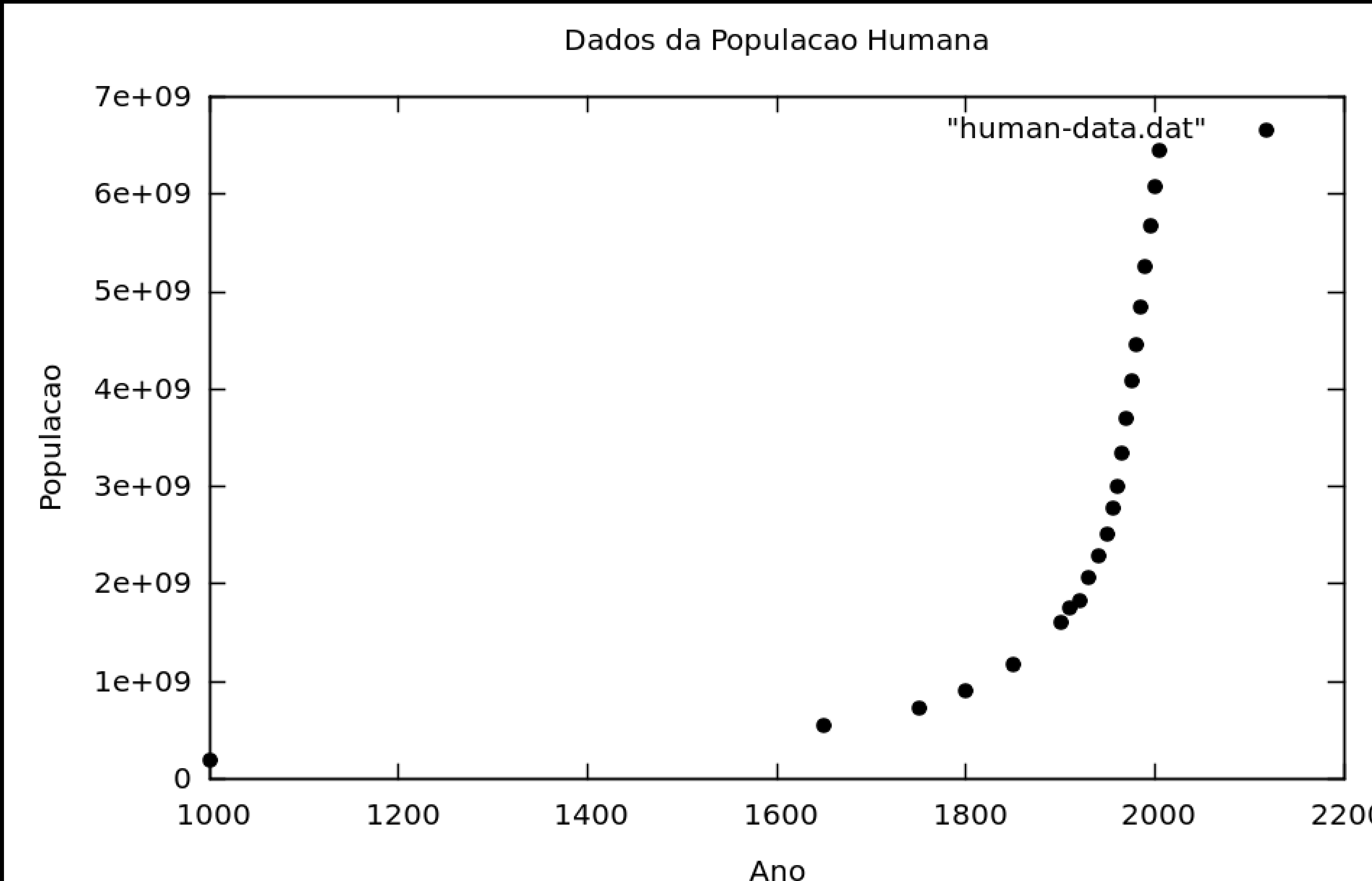
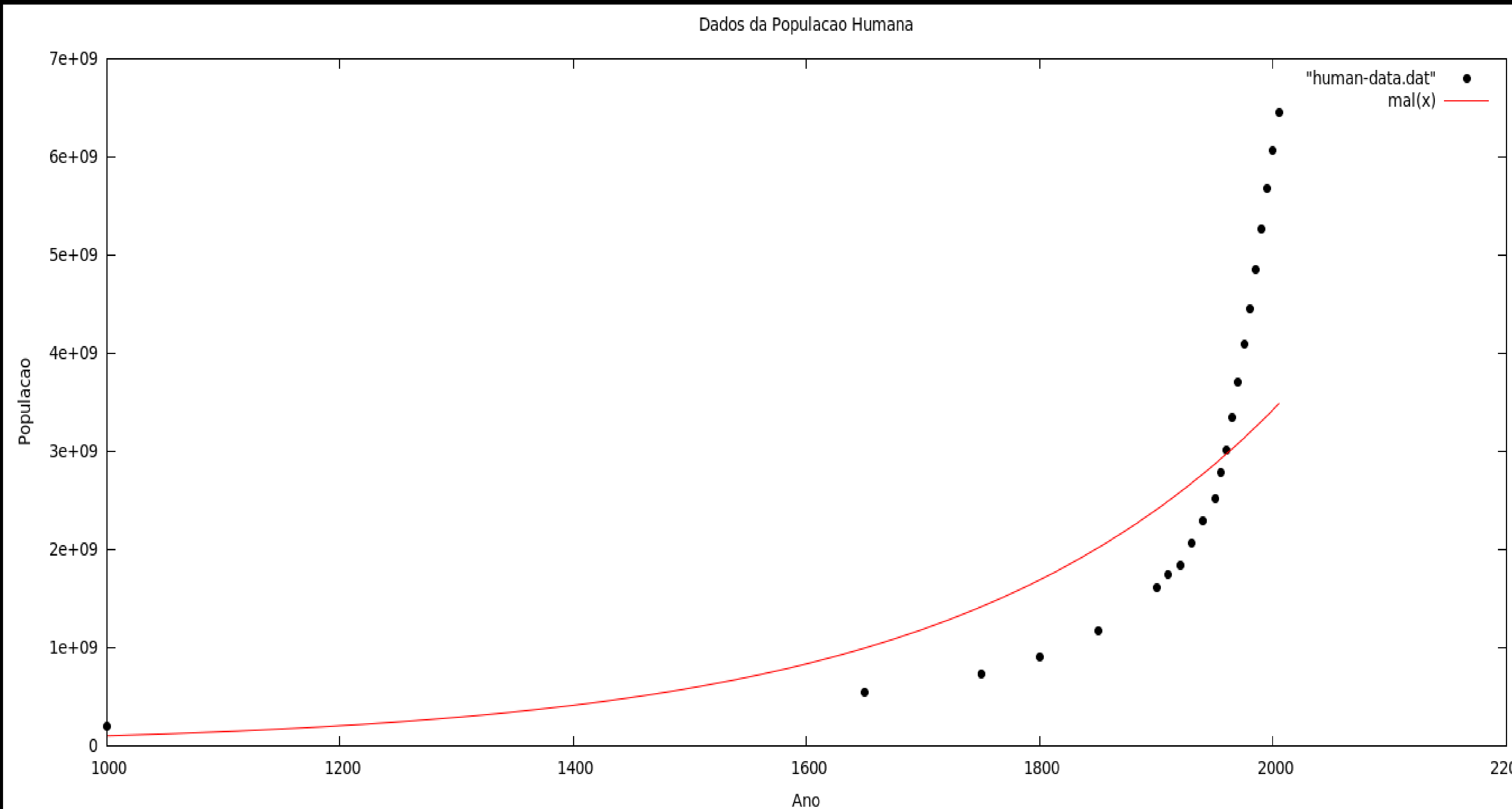
World human population.

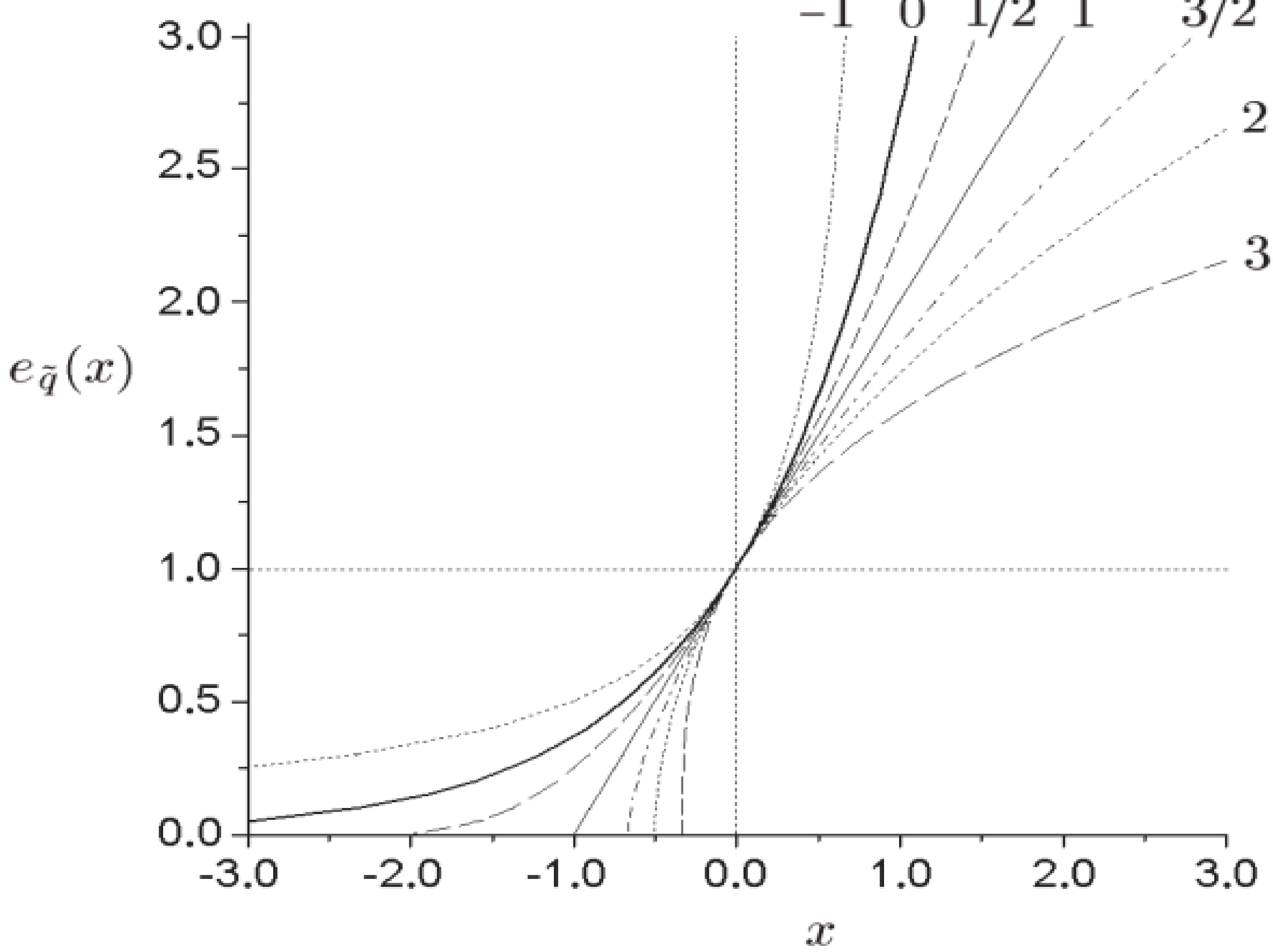
Year	Population (millions)	Year	Population (millions)
1000	200	1955	2780
1650	545	1960	3005
1750	728	1965	3345
1800	906	1970	3707
1850	1171	1975	4086
1900	1608	1980	4454
1910	1750	1985	4850
1920	1834	1990	5263
1930	2070	1995	5674
1940	2295	2000	6070
1950	2517	2005	6453



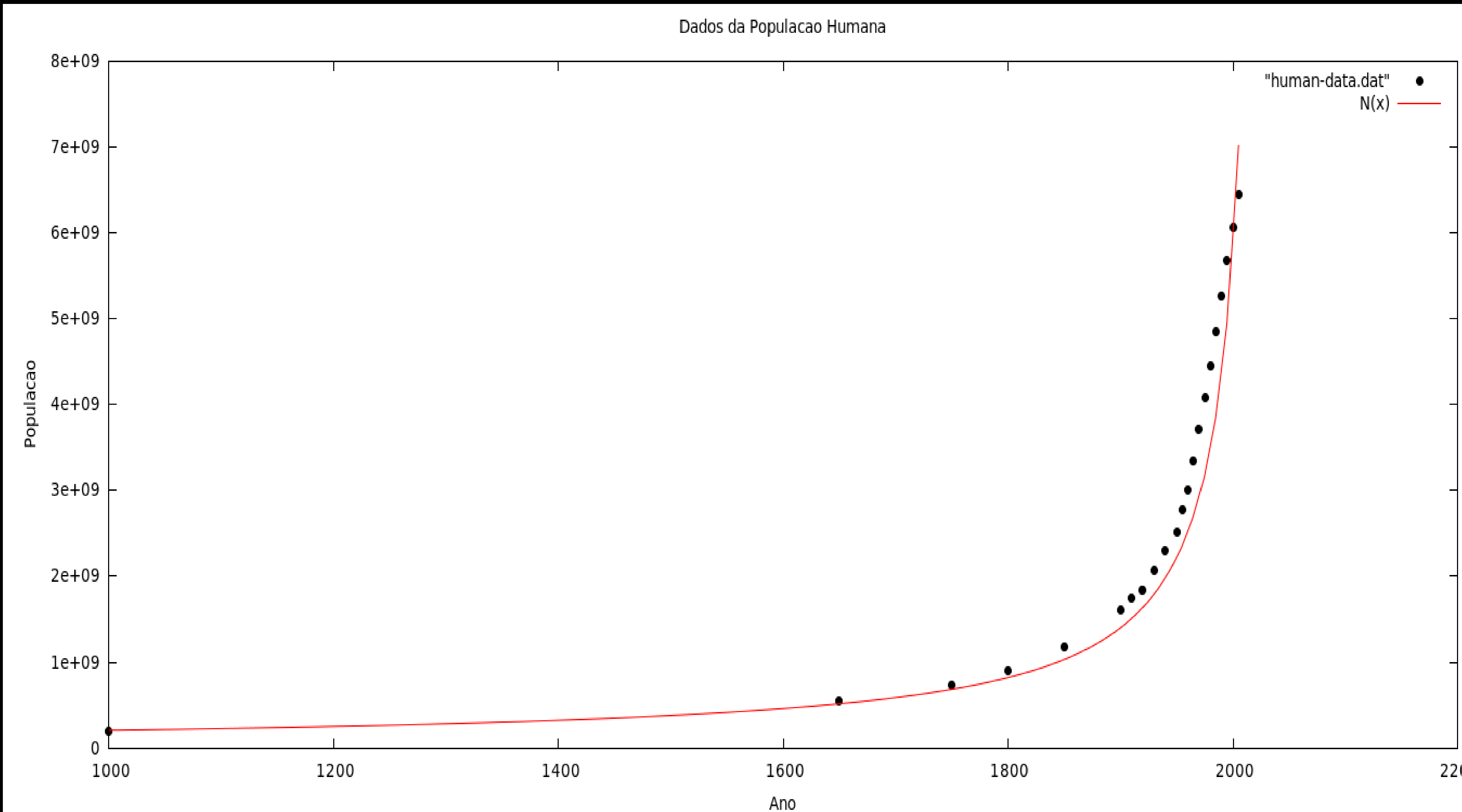
Dados da Populacao Humana
"human-data.dat"
Populacao
7e+09
6e+09
5e+09
4e+09
3e+09
2e+09
1e+09
0
1000
1200
1400
1600
1800
2000
2200
Ano



Dados da Populacao Humana
Populacao
Ano
"human-data.dat"
mal(x)
7e+09
6e+09
5e+09
4e+09
3e+09
2e+09
1e+09
0
1000
1200
1400
1600
1800
2000
2200



−1
0
1/2
1
3/2
2
3
3.0
2.5
2.0
1.5
1.0
0.5
0.0
$e_{\tilde{q}}(x)$
-3.0
-2.0
-1.0
0.0
1.0
2.0
3.0
x


Dados da Populacao Humana
"human-data.dat"
N(x)
Populacao
Ano
0
1e+09
2e+09
3e+09
4e+09
5e+09
6e+09
7e+09
8e+09
1000
1200
1400
1600
1800
2000
2200

Doomsday: Friday, 13 November, A.D. 2026

At this date human population will approach infinity if it grows as it has grown in the last two millenia.

Heinz von Foerster, Patricia M. Mora, Lawrence W. Amiot

Among the many different aspects which may be of interest in the study of biological populations (*1*) is the one in which attempts are made to estimate the past and the future of such a population in terms of the number of its elements, if the behavior of this population is observable over a reasonable period of time.

All such attempts make use of two fundamental facts concerning an individual element of a closed biological population—namely, (i) that each element comes into existence by a sexual or asexual process performed by another element of this population ("birth"), and (ii) that after a finite time each element will cease to be a distinguishable member of this population and has to be excluded from the population count ("death").

Under conditions which come close to being paradise—that is, no environmental hazards, unlimited food supply, and no detrimental interaction between elements—the fate of a biological population as a whole is completely determined at all times by reference to the two fundamental properties of an individual element: its fertility and its mortality. Assume, for simplicity, a fictitious population in which all elements behave identically (equivivant population, *2*) displaying a fertility of γ_0 offspring per element per unit time and having a mortality $\theta_0 = 1/t_m$, derived from the life span for an individual element of t_m units of time. Clearly, the elements in the population, is given by

$$\frac{dN}{dt} = \gamma_0 N - \theta_0 N = a_0 N \qquad (1)$$

where $a_0 = \gamma_0 - \theta_0$ may be called the productivity of the individual element. Depending upon whether $a_0 \gtreqless 0$, integration of Eq. 1 gives the well-known exponential growth or decay of such a population with a time constant of $1/a_0$.

In reality, alas, the situation is not that simple, inasmuch as the two parameters describing fertility and mortality may vary from element to element and, moreover, fertility may have different values, depending on the age of a particular element.

To derive these distribution functions from observations of the behavior of a population as a whole involves the use of statistical machinery of considerable sophistication (*3, 4*).

However, so long as the elements live in our hypothetical paradise, it is in principle possible, by straightforward mathematical methods, to extract the desired distribution functions, and the fate of the population as a whole, with all its ups and downs, is again determined by properties exclusively attributable to individual elements. If one foregoes the opportunity to describe the behavior of a population in all its temporal details and is satisfied

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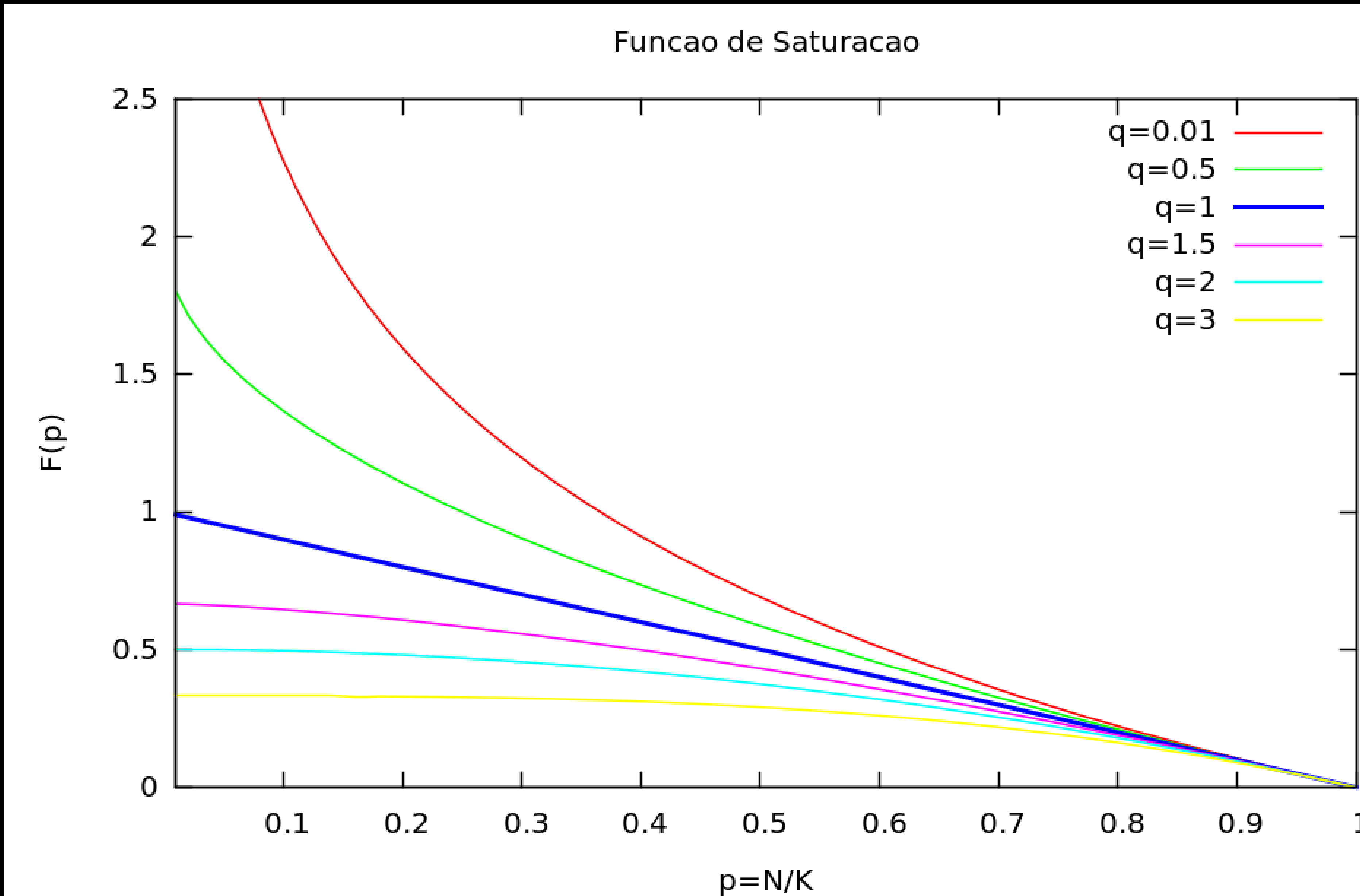
Funcao de Saturacao
2.5
2
1.5
1
0.5
0
F(p)
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
p=N/K
q=0.01
q=0.5
q=1
q=1.5
q=2
q=3